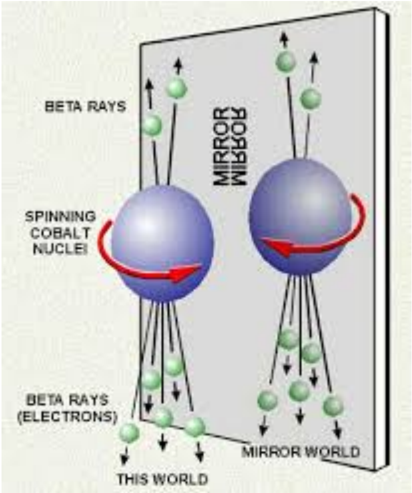
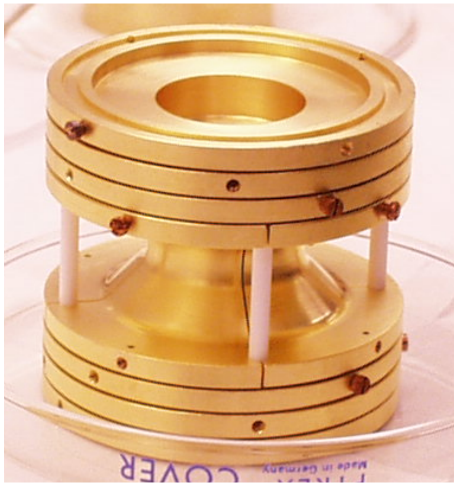
Standard Model and ion traps: symmetries galore





Jason Clark

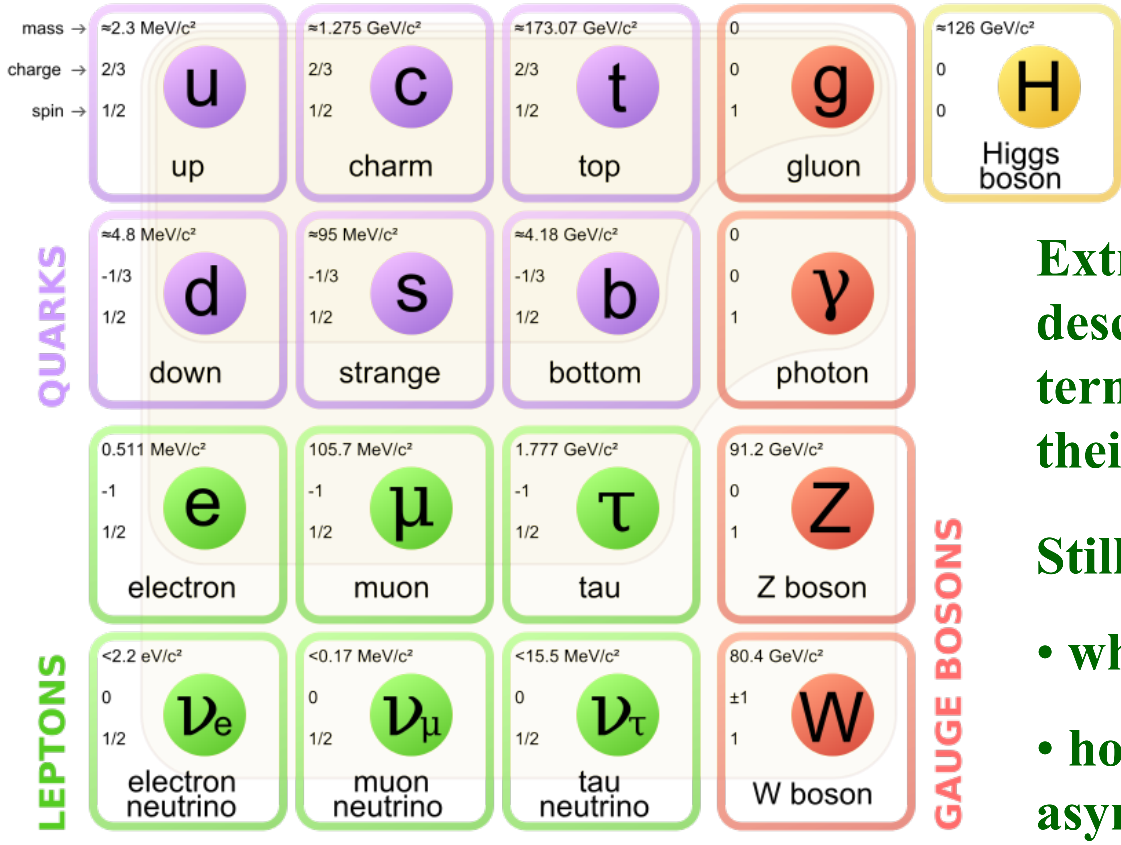
Exotic Beam Summer School

July 28 – August 1, 2014

Lecture 2 outline

- Overview of the Standard Model (SM)
- Nature of the weak interaction and β decay
- Tests of the SM description of the weak interaction
  - β-ν correlations
  - unitarity of CKM matrix
  - constancy of *Ft* values
  - searches for neutrinoless double β decay
- Methods used to test SM description of weak interaction
  - Paul trap
  - Penning trap

Standard Model



Extremely successful at describing the world around us in terms of basic constituents and their interactions.

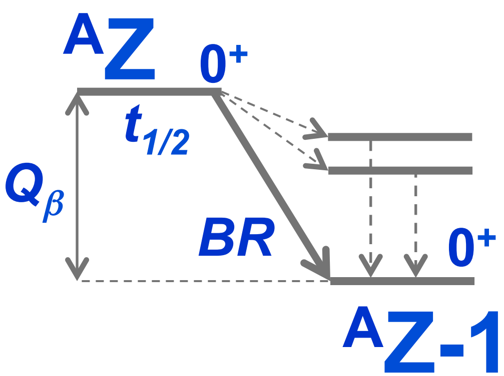
Still some open questions:

- **what is dark energy?**
- **how did the matter/antimatter asymmetry come to be?**

Is there something beyond the Standard Model?

Tests of the Standard Model: Superallowed $0^+ \rightarrow 0^+$ Fermi β decay

The nucleus is a complex quantum system not generally amenable to an exact description. However, by a proper choice of nuclear system, specific physical processes can be isolated and determined to high precision. (Ex: Superallowed 0+ → 0+ Fermi β decay)



Relationship between partial halflife, matrix element, and phase space:

$$\frac{t_{1/2}}{BR} = \frac{K}{G_V^2 \cdot |M_F|^2 \cdot f(Z, Q_\beta)}$$

Rearranging gives "ft value":

$$ft = \frac{K}{2G_V^2} \qquad |M_F|^2 = 2$$

For $0^+ \rightarrow 0^+$ decays

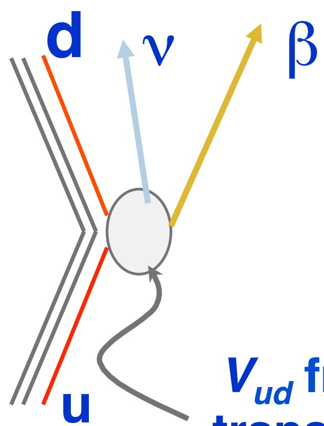
And including a bunch of small-ish corrections:

$$Ft \equiv ft\underbrace{(1+\delta_R')}_{\sim 1.5\%}\underbrace{(1+\delta_{NS}-\delta_C)}_{0.3\text{-}1.5\%} = \frac{K}{2G_V^2\underbrace{(1+\Delta_R^V)}_{2.4\%}}$$

Many things we can investigate after studying several different nuclei

***Ft* constant for $0^+ \rightarrow 0^+$ decays?**

Conserved vector current in weak interaction



symmetry between EM and weak interaction → conservation of vector part of weak interaction (like conservation of charge)

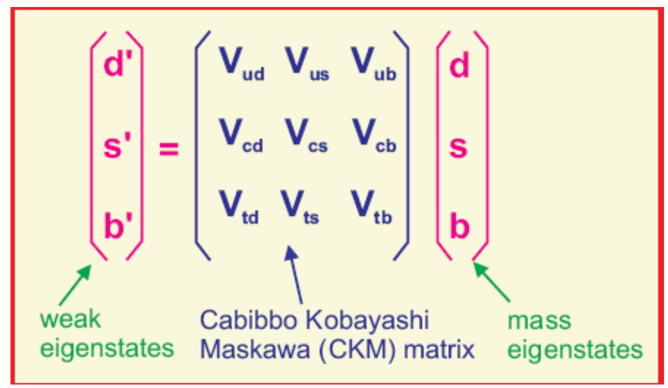
V_{ud} from u→d transformation

Value of *Ft*?

Test unitarity of CKM matrix

Determine G_V^2 and compare to purely leptonic μ decay (G_μ^2) to extract V_{ud} matrix element

$$V_{ud}^2 = \frac{G_V^2}{G_\mu^2}$$



$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 1?$$

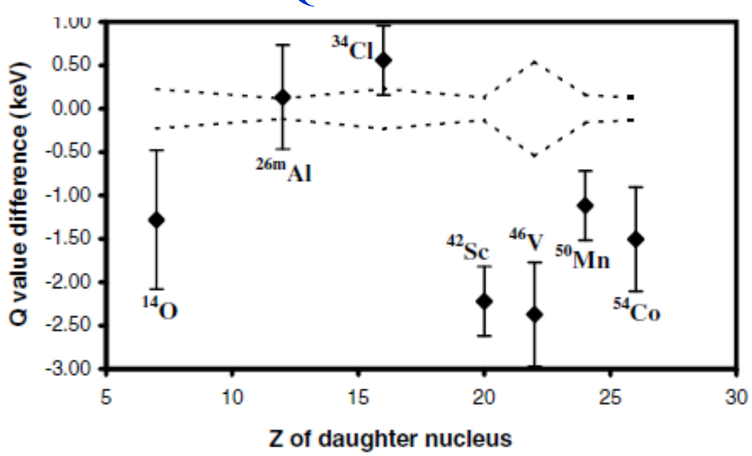
Penning traps improve precision of previously-studied isotopes and provide access to new ones

Ft **value depends strongly on** $\boldsymbol{Q_b}$

$$f\left(Z, Q_\beta\right) \sim Q_\beta^5$$

Penning trap measurement of ^{46}V uncovered systematic shift in previous data from (^{3}He,t) measurements

Difference between earlier (^{3}He,t) measurements of Q value and modern data



G. Savard *et al.*, PRL 95, 102501 (2005)

Penning traps allow measurement of isotopes where both parent and daughter are radioactive

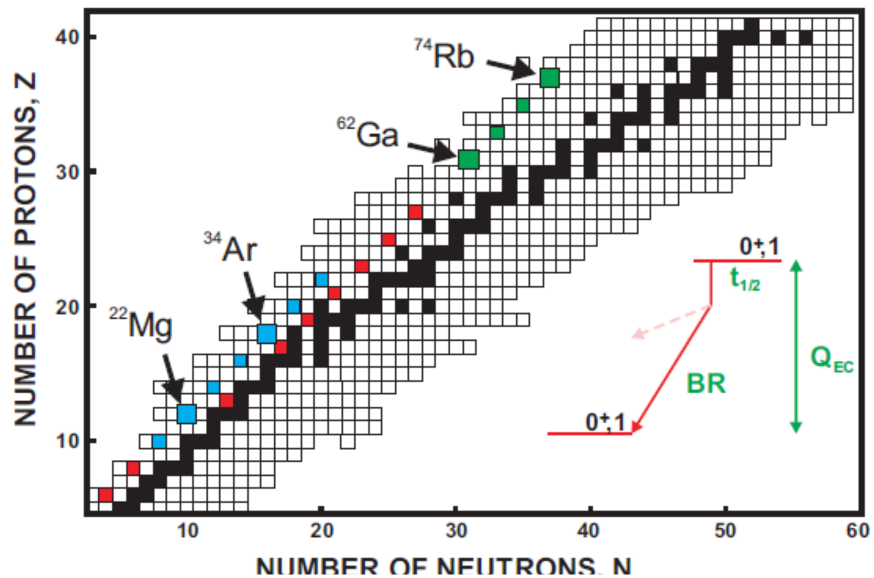
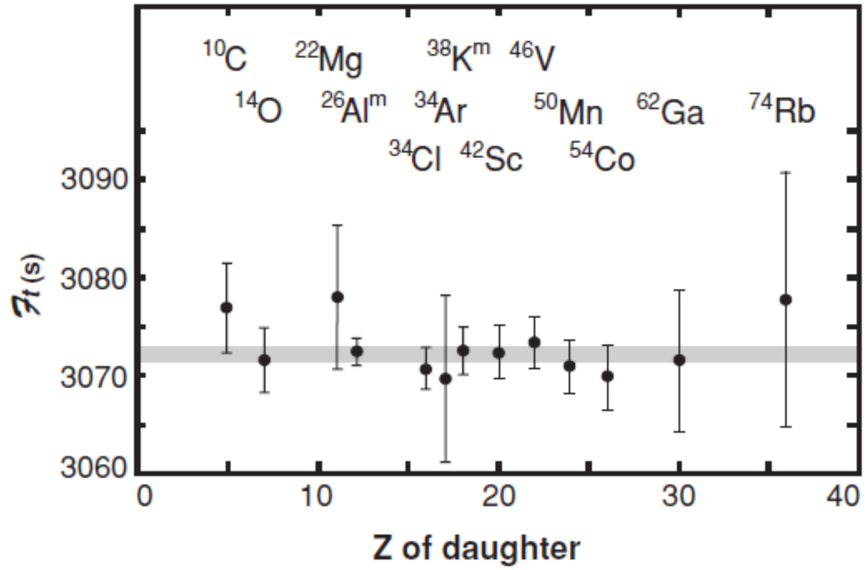


Figure from J.C. Hardy

Results and impact on weak interaction physics



<u>Constancy of *ft* values:</u>

Values constant to ±0.4% for all nuclei → conservation of the weak C_V current (like EM current)

<u>Determine V_{ud} matrix element:</u>

$|V_{ud}|^2 = 0.94916 \pm 0.00044$

And when combined with $|V_{us}|^2+|V_{ub}|^2$, gives the unitarity test:

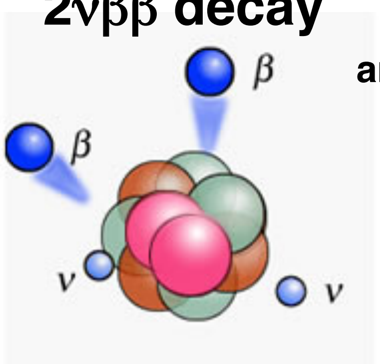
$|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2 = 0.99995 \pm 0.00061$

J.C. Hardy and I.S. Towner, PRC 79, 055502 (2009). Towner & Hardy, Rep. Prog. Phys. 73, 046301 (2010).

Are neutrinos their own antiparticle? What is the neutrino mass scale and hierarchy?

Only known way to determine this is neutrinoless double beta decay ($0\nu\beta\beta$ decay). In principle, this is a clear signature… but extremely sensitive techniques are required

"ordinary" $2\nu\beta\beta$ decay

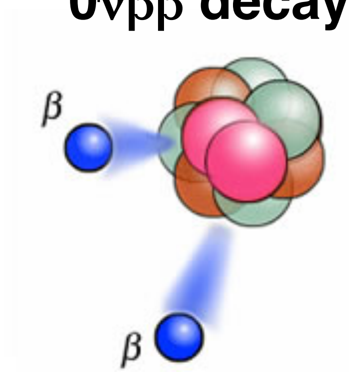


Equal numbers of matter (β^-) and antimatter ($\bar{\nu}$) emitted

Broad β energy spectrum (neutrinos escape detection)

Observed: $T_{1/2}$~10^{21} years

$0\nu\beta\beta$ decay



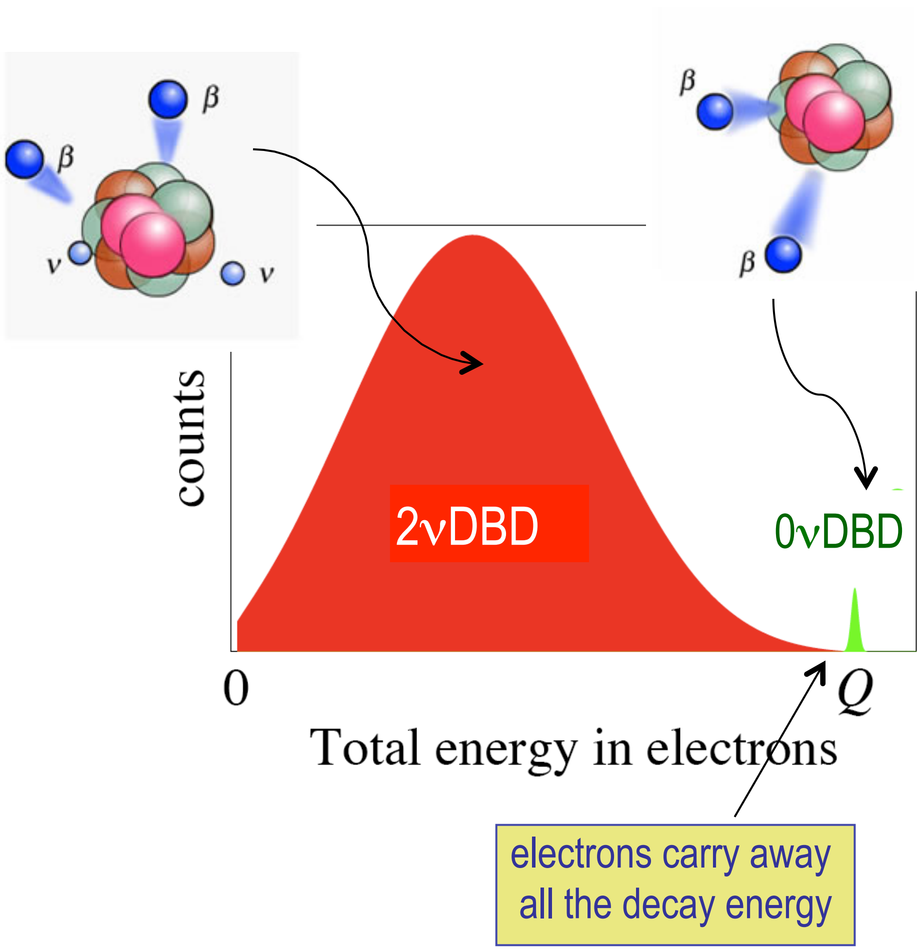
Only matter (β^-) emitted

Signal at total decay energy (all energy detected)

Recognized in 1930s… but never been observed: $T_{1/2}$>10^{25} years

A mechanism for matter/antimatter asymmetry

Penning traps precisely determine where to search for very tiny signal



Penning traps have pinned down Q values for the isotopes to <1 keV

^{76}Ge

B.J. Mount *et al*., PRC **81**, 032501 (2010)
S. Rahaman *et al*., PLB **662**, 111 (2008)
G. Douysset *et al*., PRL **86**, 4259 (2001)

^{130}Te

S. Rahaman *et al*., PLB **703**, 412 (2011)
N.D. Scielzo *et al*., PRC **80**, 025501 (2009)
M. Redshaw *et al*., PRL **102**, 212502 (2009)
D.A. Nesterenko *et al*., PRC **86**, 044313 (2012)

^{82}Se

D.L. Lincoln *et al*., PRL **110**, 012501 (2013)

^{150}Nd

V.S. Kolhinen *et al*., PRC **82**, 022501 (2010)

^{48}Ca

M. Redshaw *et al*., PRC **86**, 041306 (2012)

+ others

Confining ions with both electric and magnetic fields ...

the Penning trap

F.M. Penning

H. G. Dehmelt

The ‘not so easy’ way to confine ions in 3D

Taking the general form of 1D confinement and extending it to 3D:

$$V = \lambda_x x^2 + \lambda_y y^2 + \lambda_z z^2$$

Result it that:

$$\lambda_x = \lambda_y = -\frac{1}{2}\lambda_z$$

$$\Rightarrow V = -\lambda(x^2 + y^2 - 2z^2)$$

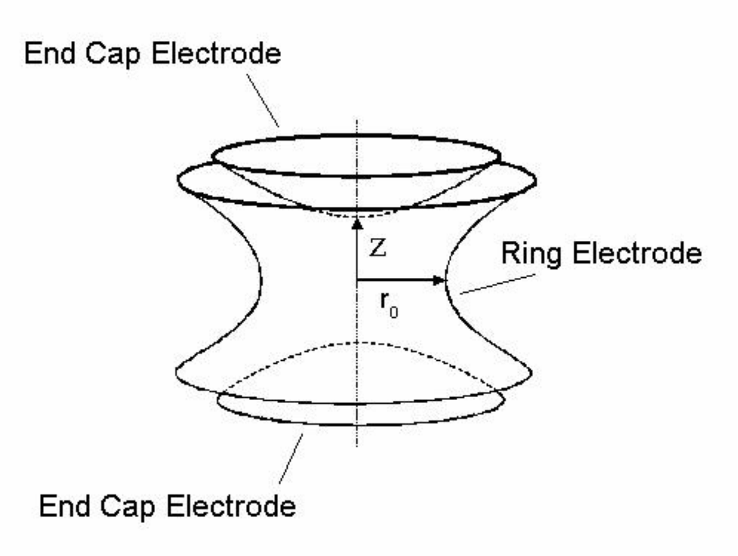
Or in cylindrical coordinates:

$$\Rightarrow V = -\lambda(r^2 - 2z^2)$$

The 'not so easy' way to confine ions in 3D

If we define: $V = \frac{\varphi_o}{2}$ at $(r,z) = (\pm r_o, 0)$

then: $\lambda = -\frac{\varphi_o}{2r_o^2}$



Result is equipotential surfaces:

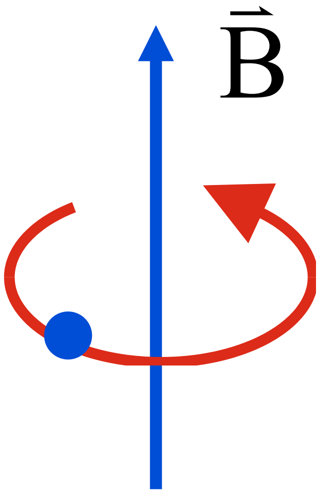
$$\frac{r^2}{r_o^2} - \frac{2z^2}{r_o^2} = 1 \quad \text{with} \quad V = \frac{\varphi_o}{2}$$

$$\frac{r^2}{r_o^2} - \frac{2z^2}{r_o^2} = -1 \quad \text{with} \quad V = -\frac{\varphi_o}{2}$$

Dimensional constraint:

$$\left(\frac{r_o}{z_o}\right)^2 = 2$$

Ions within a magnetic field



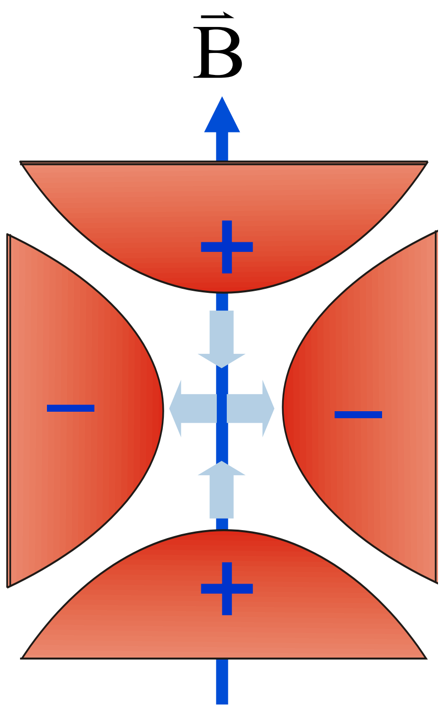
- **constant axial magnetic field**
- **particle orbits in horizontal plane with cyclotron frequency:**

$$\omega_c = \frac{qB}{m}$$

- **free to escape axially**

Confine ions by adding an electric field

Add a harmonic potential (along magnetic field axis) to confine particles.

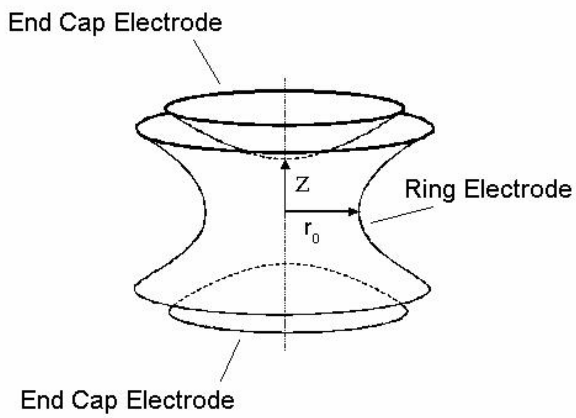


Confining potential:

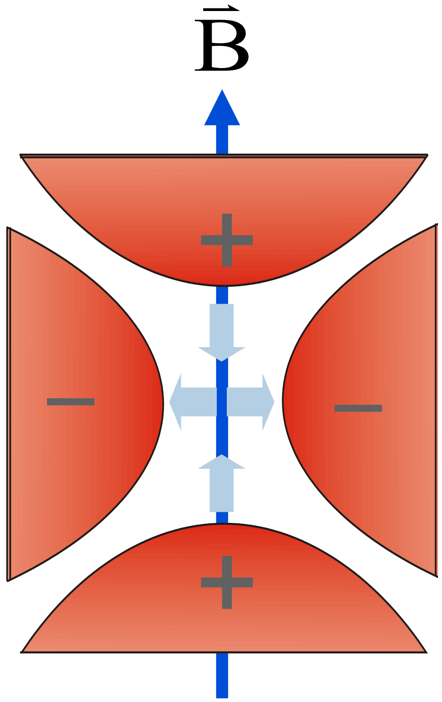
$$V = \frac{V_o}{2d^2}\left(z^2 - \frac{r^2}{2}\right)$$

Characteristic trap dimension:

$$d = \sqrt{\frac{1}{2}\left(z_o^2 + \frac{r_o^2}{2}\right)}$$



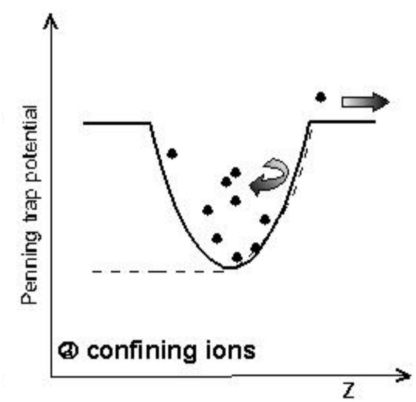
Motion of ions in a Penning trap



Equations of motion: $$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Along magnetic field axis: $$m\ddot{z} = -\frac{qV}{d^2} z$$

Ions undergo oscillations with frequency: $$\omega_z = \sqrt{\frac{eV}{md^2}}$$



Motion of ions in a Penning trap

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Radial equations of motion:

$$\ddot{x} = \frac{\omega_z^2}{2} x + \omega_c \dot{y}$$

$$\ddot{y} = \frac{\omega_z^2}{2} y - \omega_c \dot{x}$$

Effect of the electric field is to split the radial motion into two components:

$$\omega_c \rightarrow \omega_+ , \quad \omega_-$$

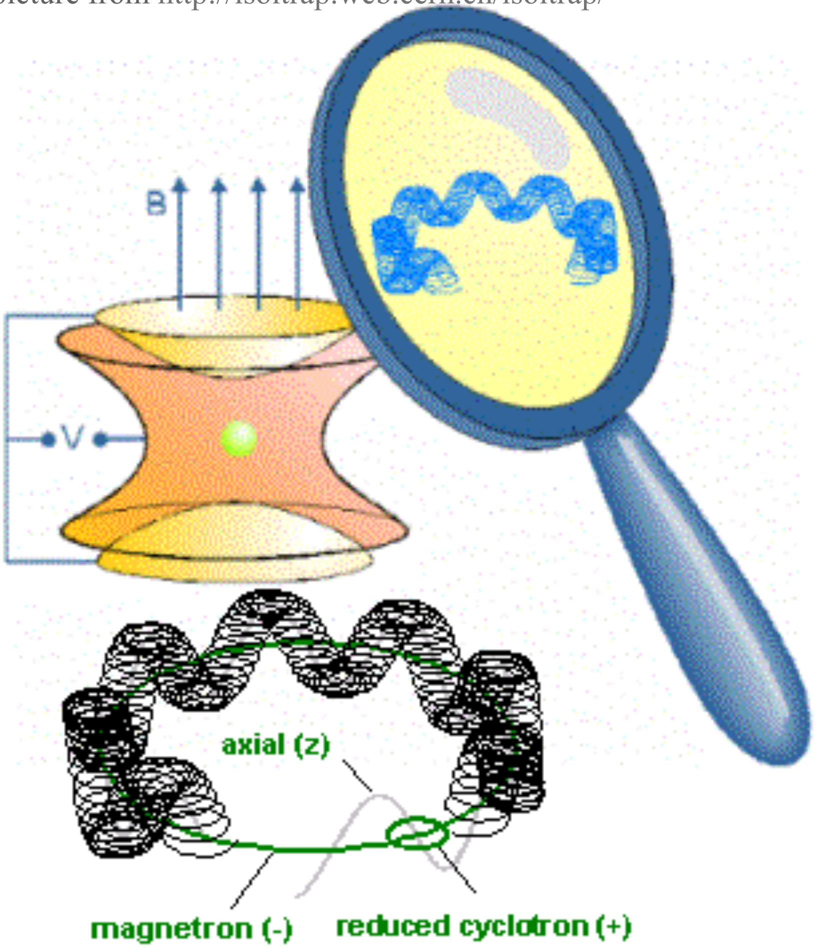
ω_+: reduced cyclotron motion

ω_-: magnetron motion

$$\omega_\pm = \frac{\omega_c}{2} \pm \sqrt{\frac{\omega_c^2}{4} - \frac{\omega_z^2}{2}}$$

Putting it all together … the frequency hierarchy

picture from http://isoltrap.web.cern.ch/isoltrap/



Frequency relations:

$$\omega_c^2 = \omega_+^2 + \omega_-^2 + \omega_z^2$$

$$\omega_c = \omega_+ + \omega_-$$

Frequency hierarchy:

$$\omega_- << \omega_z << \omega_+ \sim \omega_c$$

A u	ν_z	ν_-	ν_+
1	517 kHz	1475 Hz	91 MHz
10	163 kHz	1475 Hz	9.1 MHz
40	82 kHz	1476 Hz	2.2 MHz
80	58 kHz	1477 Hz	1.1 MHz
115	48 kHz	1478 Hz	787 kHz
190	38 kHz	1480 Hz	475 kHz
250	33 kHz	1481 Hz	361 kHz

Frequency values for singly charged ions in a Penning trap with parameters r_0=14.4 mm, z_0=8.9 mm, U_0=10 V, and B=5.9 T, as functions of mass number A.

Determining mass of ions in a Penning trap

Since:

$$\omega_c = \omega_+ + \omega_- = \frac{qB}{m}$$

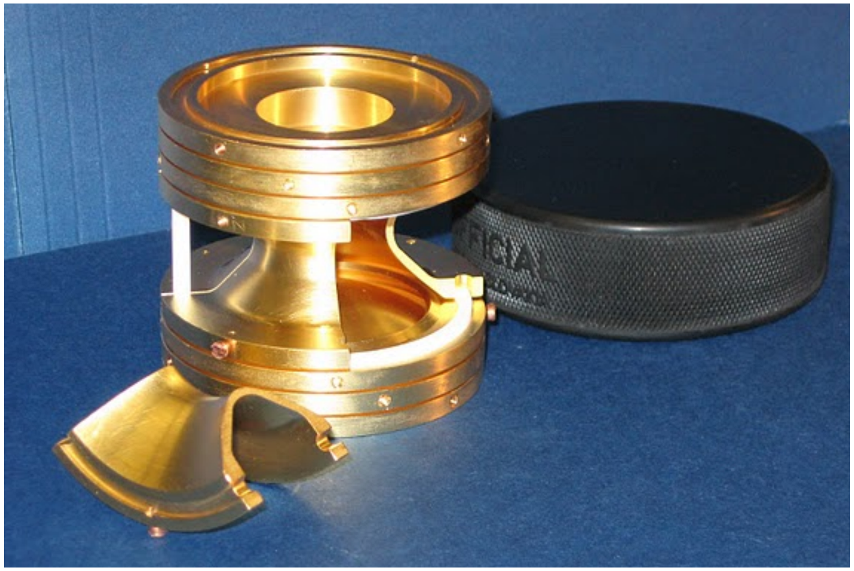
ω_c depends only on:

- the mass
- the magnetic field
- not on the electric fields

So if we can find a stable, uniform magnetic field, we can use ω_c to make accurate and precise mass measurements.

The solution: use a superconducting magnet

The Canadian Penning trap



Hockey puck

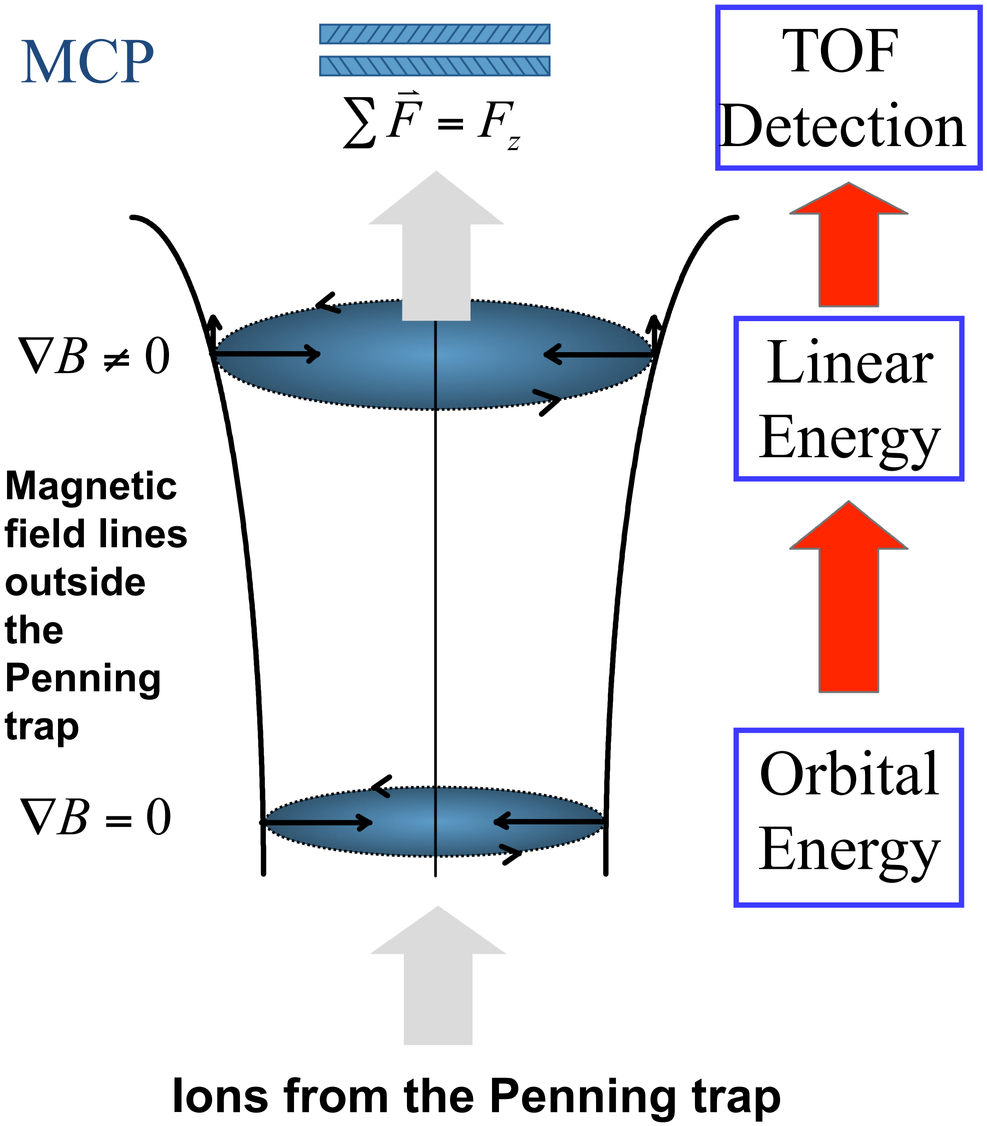
Cigarette

Detection of the energy gained from frequency conversion

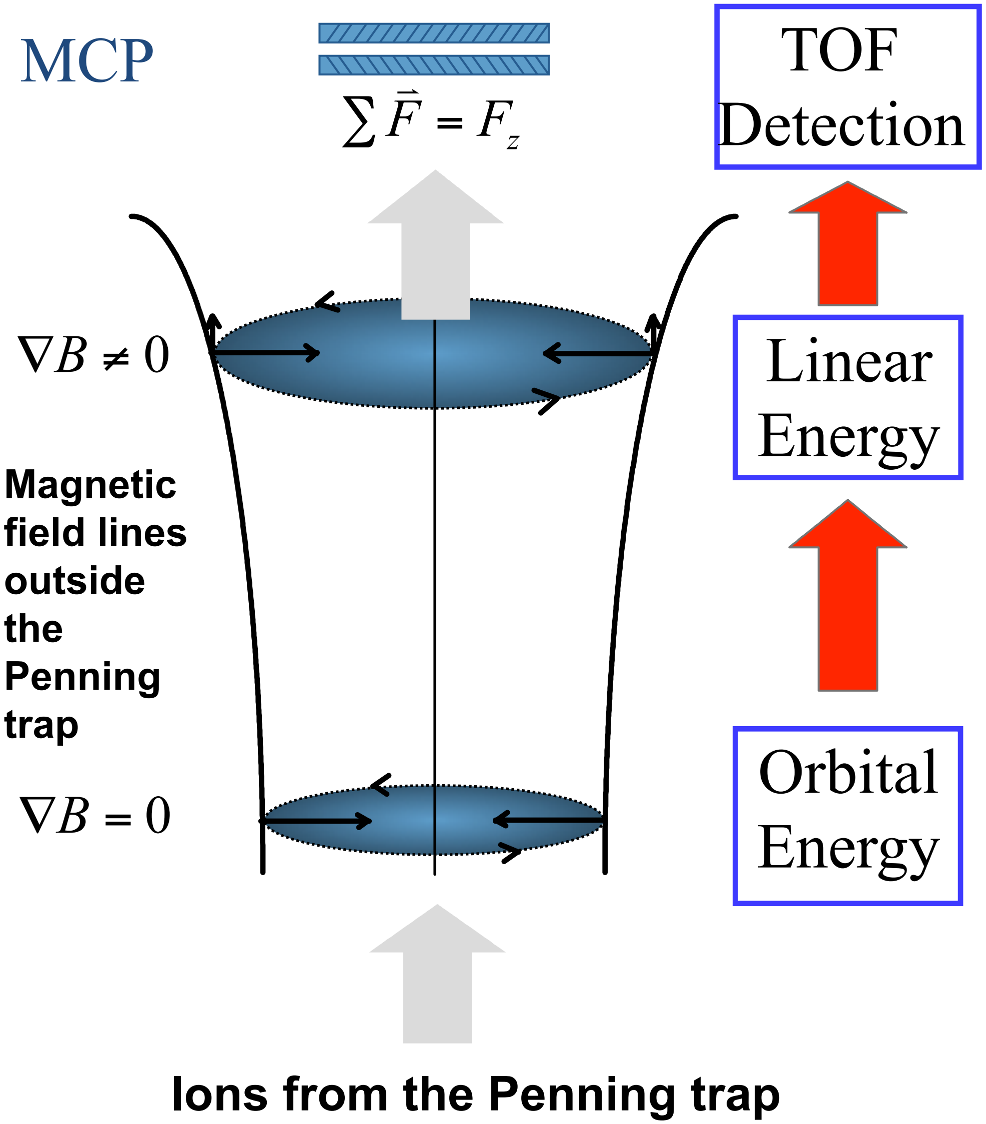
- After excitation, ions are ejected from the trap and guided toward a microchannel plate ion detector

- Magnetic field gradient outside the Penning trap converts any radial energy into axial energy

- Higher energy ions arrive with shorter time-of-flight

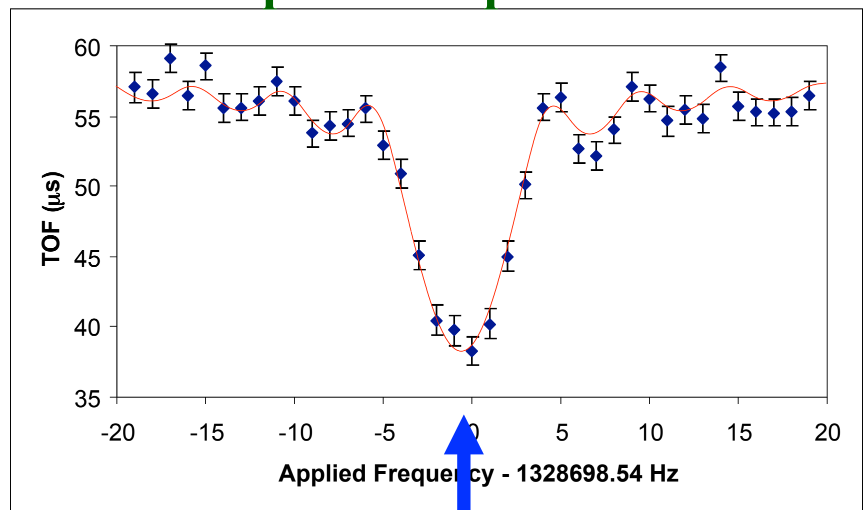


Determining the cyclotron frequency



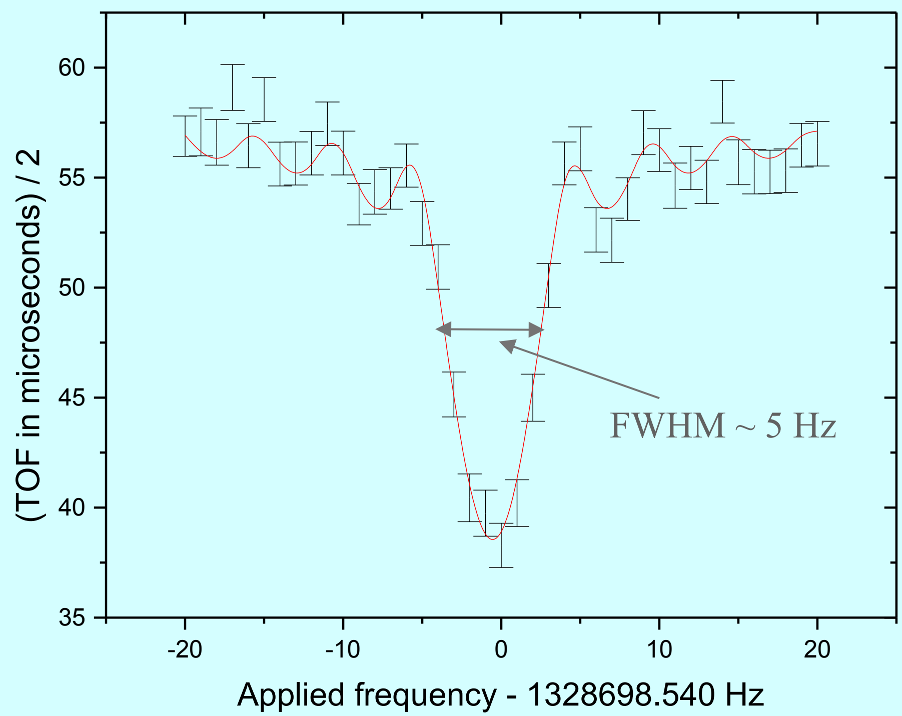
- **Perform frequency scan where each new bunch of trapped ions is subjected to a different frequency**
- **Repeat frequency scan many times to get statistics**

Sample TOF spectrum:



$$\omega_c = \frac{q_c B_c}{m_c}$$

Sample time-of-flight (TOF) spectrum



$$\omega_c = \frac{q_c B_c}{m_c}$$

Unknown: $$\omega_? = \frac{q_? B_c}{m_?}$$

$$\frac{Unknown}{Calibration} \Rightarrow m_? = \frac{q_?}{q_c} \frac{\omega_c}{\omega_?} m_c$$

Well-known mass is a requirement for accurate measurements.

What to use as a calibrant?

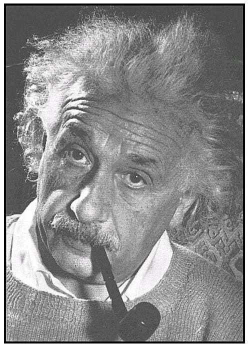
Want to use a well-known mass.

Most precise masses:

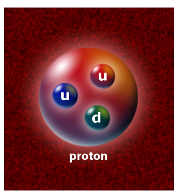
^{12}C:	$\Delta m = 0$
^{4}He:	$\Delta m = 0.06$ eV
^{1}H:	$\Delta m = 0.09$ eV
^{16}O:	$\Delta m = 0.15$ eV

Generally, we use compounds of ^{1}H and ^{12}C: masses are known precisely and hydrocarbons cover essentially all masses.

Energy and mass and precision



$$E = mc^2$$



Proton: $E = (1.67 \cdot 10^{-27}\ \text{kg})(3 \cdot 10^{8}\ \text{m/s})^2$

$= 1.50 \cdot 10^{-10}\ \text{J}$

$= 0.939\ \text{GeV}$

me: $E = (72.5\ \text{kg})(3 \cdot 10^{8}\ \text{m/s})^2$

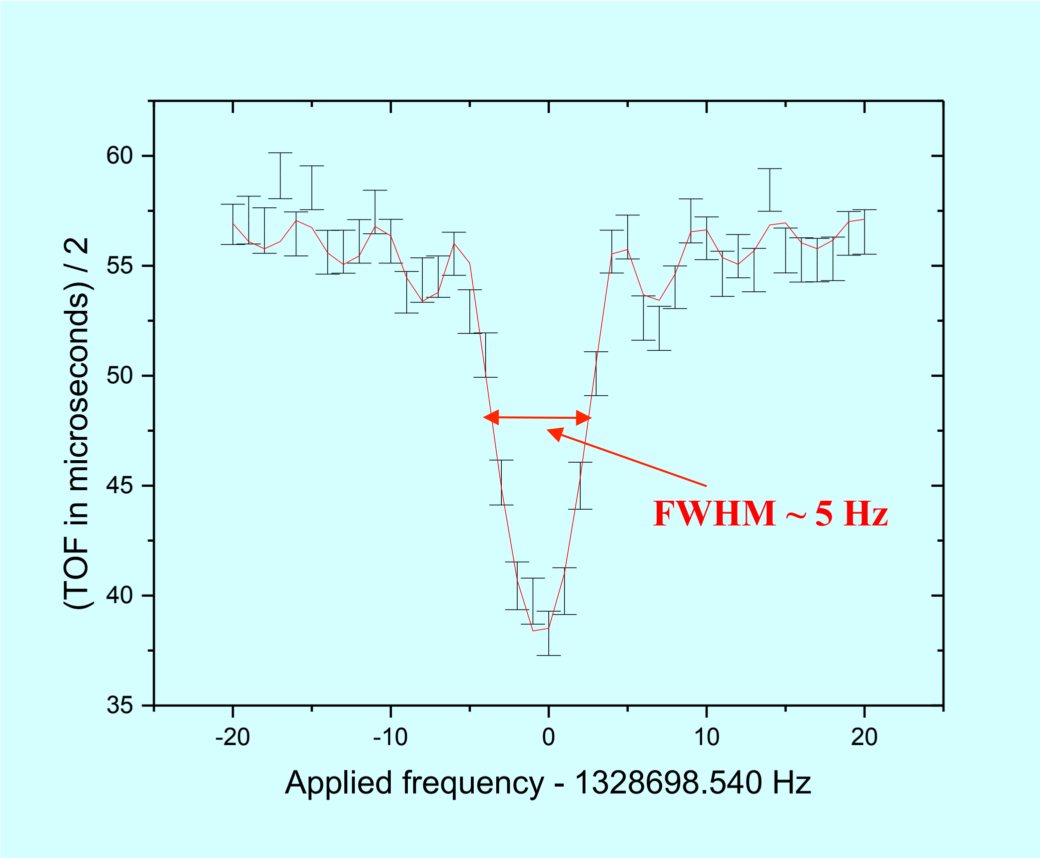
$= 40\ \text{EEeV}$

For A=100, mass precision obtained ~ 300 eV => $3\text{x}10^{-9}$ precision

That's like measuring my mass to 0.0000002 kg!

Sample TOF spectra

Calibration: $C_5 H_8$



$$\text{Resolution} = \frac{\Delta\omega_{FWHM}}{\omega}$$

$$\Delta\omega_{FWHM} \sim \frac{0.9}{T_{conv}}$$

$$\text{Precision}, \frac{\Delta \text{m}}{\text{m}} \propto \frac{m}{T_{conv} qB\sqrt{N}}$$

Increasing mass precision

$$\text{Precision,} \quad \frac{\Delta \mathrm{m}}{\mathrm{m}} \propto \frac{m}{T_{conv}\, q B \sqrt{N}}$$

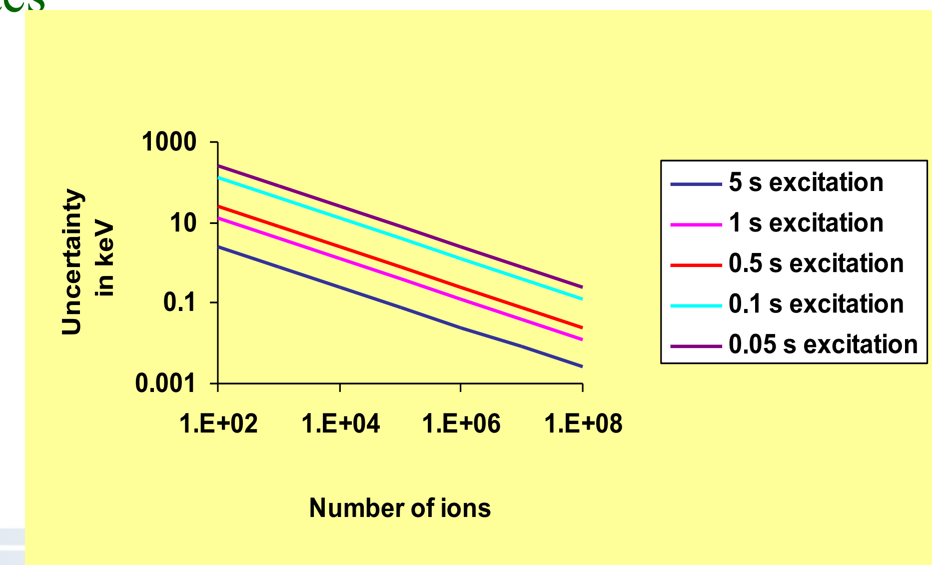
Limited by half-life of nuclide

Higher charge states

Stronger fields

Statistics

For measurements on short-lived isotopes the limiting factors are T_{CONV} and N.



Summary

- **Standard Model is a robust model that describes basic constituents of nature and the interactions between them**
  - **yet still unresolved issues (matter-antimatter asymmetry, dark energy, …)**
- **Low energy tests of SM description of weak interaction (β decay) have been quite fruitful**
  - **β-ν correlations**
  - **unitarity of CKM matrix**
  - **constancy of Ft values**
  - **searches for neutrinoless double β decay**
- **Ion traps becoming one of the most used tools for precision tests of SM model:**
  - **Paul trap**
  - **Penning trap**
- **Potentially many new discoveries/advances early in your career!**

Questions?