The foundational models of the nucleus were developed based on structure studies of stable and near-stable nuclei.

Far-from-stability studies of nuclear structure can test these models in regions of proton/neutron number far away from their point of origin.

LECTURE 3: Simple (?) structures in nuclei
--seniority; quadrupole collectivity

• Illustration of the two types of nuclear structure on which there is consensus (?)
  
  seniority (pair-dominated) structures
  quadrupole (shape-dominated) structures

• Illustration of isomer and Coulex spectroscopy
Excited $0^+$ states at closed shells: mixing and repulsion of pair configurations in $^{90}$Zr

N=50: $g_{9/2}$ seniority structure

$E(2_1^+)$: in some cases it appears high, implying a closed (sub)shell, but is due to a depression of the ground-state energy

<table>
<thead>
<tr>
<th>State</th>
<th>Energy (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^+$</td>
<td>1761</td>
</tr>
<tr>
<td>$0^+$</td>
<td>$p_{1/2}^2$</td>
</tr>
<tr>
<td>$0^+$</td>
<td>$g_{9/2}^2$</td>
</tr>
<tr>
<td>$0^+$</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Energy (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(p_{1/2}g_{9/2})^2$</td>
<td>3589</td>
</tr>
<tr>
<td>$(p_{1/2}^2g_{9/2})_0$</td>
<td>2760</td>
</tr>
<tr>
<td>$(p_{1/2}^2g_{9/2})^4$</td>
<td>2644</td>
</tr>
<tr>
<td>$(p_{1/2}^2g_{9/2})^6$</td>
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</tr>
<tr>
<td>$2^+$</td>
<td>1509</td>
</tr>
<tr>
<td>$2^+$</td>
<td>1431</td>
</tr>
<tr>
<td>$0^+$</td>
<td>0</td>
</tr>
</tbody>
</table>

$^{90}$Zr$_{50}$  $^{92}$Mo$_{50}$  $^{94}$Ru$_{50}$  $^{96}$Pd$_{50}$
Neutron-rich Ni isotopes probably dominated by a \( \nu_{9/2} \) seniority structure

The relatively high \( E(2^+_1) \) value in \( ^{68}\text{Ni} \) is probably not indicative of a doubly closed shell

\( ^{68}\text{Ni} \) has a structure which is similar to \( ^{90}\text{Zr} \), i.e., a seniority structure that is dominated by \( g_{9/2} \) and \( p_{1/2} \); but the additional states indicate that \( f_{5/2} \) and \( p_{3/2} \) (and, possibly, proton pair excitations* across the \( Z = 28 \) shell) are active.

*See D. Pauwels et al., PR C82, 027304 (2010)
The $g_{9/2}$ seniority structure persistence near the doubly closed shell $^{78}\text{Ni}$, $^{100}\text{Sn}$, and $^{132}\text{Sn}$ nuclei

Figure taken from A. Jungclaus et al., PRL 99 132501 (2007)

The $8^+$ states are isomeric:

- $^{76}\text{Ni}$: $T_{1/2}$ (ns) = 590, $E_\gamma$ (keV) = 144, $B(E2)$ W.u. = 0.71
- $^{98}\text{Cd}$: $T_{1/2}$ (ns) = 480, $E_\gamma$ (keV) = 147, $B(E2)$ W.u. = 0.46

In $^{130}\text{Cd}$ the ordering of the 128/138 keV $\gamma$-ray cascade is not certain.

The constancy of the $B(E2)$ values, independent of whether the structures are dominated by protons or neutrons and independent of mass, is remarkable and shows the simple nature of seniority structures.
Neutron-rich Sn isotope structure dominated by $\nu h_{11/2}$ seniority structure
$^{152}\text{Sm}: \text{what is the nature of the } 0_2^+ (685 \text{ keV}) \text{ state?}$

We all thought that the $0_2^+ (685 \text{ keV})$ state in $^{152}\text{Sm}$ was the classic example of a $\beta$ vibration.
Shape coexistence in the N = 90 isotones: explains the E0 transition strengths


\[ \rho^2 \cdot 10^3 = \alpha^2 \beta^2 \left( \Delta \langle r^2 \rangle \right)^2 \cdot 10^3 \frac{Z^2}{R_0^4} \]

\[ R_0 = 1.2A^{1/3} \text{ fm} \]

\( E0 \) strength is a function of mixing.
Multi-Coulex of $^{152}$Sm $0^+_2$(685 keV): strongest response is to head of K=2$^+$ band at 1769 keV.


Gammasphere + CHICO @ 88” (Berkeley)

in-band response attenuated by 99.7% decay out @ 811 level
Shape coexistence in the N = 90 isotones: coexisting K = 2 bands revealed by E0 transitions

3⁺, K = 2 → 3⁺, K = 2: 631 keV transition in $^{158}$Er has no observable γ-ray strength, only ce’s are observed -- accidental cancellation of E2; M1 is very weak.

Kulp, Wood, Garrett, Zganjar and others
$^{152}\text{Sm}$ and the neighboring $N = 90$ isotones are a manifestation of shape coexistence.

Proton particle-hole excitations across the $Z = 64$ gap may be the source of the coexisting shapes.

Less-deformed $2h$ and more-deformed $2p$-$4h$ structures coexist at low energy at $N=90$.

Strong mixing obscures the energy differences that are indicative of different shapes.

Strong $E0$ transitions are a key signature of the mixing of coexisting structures.

As observed, the $K=2$ bands will also mix strongly, resulting in $E0$ transitions.
Ground state properties, $S_{2n}$ and $\delta <r^2>$, in the regions of $N = 60, 90$ are very similar

Figure from S. Naimi et al. Phys. Rev. Lett. 105 032502 (2010)

Figure from Heyde & Wood
Universal rotor B(E2)’s

Figure from Heyde & Wood

\[ B(E2; 2 \rightarrow 0) = \frac{1}{16\pi} (Q_0^{(e)})^2, \]
\[ B(E2; I \rightarrow I - 2) = \frac{15I(I - 1)}{2(2I - 1)(2I + 1)} B(E2; 2 \rightarrow 0) \]

\[ E_I = E_0 + \frac{\hbar^2}{2\mathcal{S}} I(I + 1) \]
Multistep Coulomb excitation of $^{74,76}$Kr using radioactive beams of Kr on a $^{208}$Pb target

$^{72,74}\text{Kr}$: $O_2^+$ states observed by conversion electron spectroscopy via IT decay of $^{72m,74m}\text{Kr}$

Horizontal dashed lines show energies ($\Delta$) of unmixed $0^+$ configurations

$\rho^2 (E0) \cdot 10^3$  

<table>
<thead>
<tr>
<th>72</th>
<th>74</th>
<th>76</th>
<th>78</th>
</tr>
</thead>
<tbody>
<tr>
<td>72</td>
<td>85</td>
<td>79</td>
<td>47</td>
</tr>
</tbody>
</table>

E. Bouchez et al., PRL 90, 082502 (2003)

J.L. Wood et al. NP A651 323 (1999)
Quadrupole shape invariants constructed from E2 matrix elements for $^{74,76}$Kr

\[ \langle q^2 \rangle = \langle 0^+_1 \| \hat{Q} \| 2^+_1 \rangle \langle 2^+_1 \| \hat{Q} \| 0^+_1 \rangle + \langle 0^+_1 \| \hat{Q} \| 2^+_2 \rangle \langle 2^+_2 \| \hat{Q} \| 0^+_1 \rangle \]

for the ground state

\[ \langle q^3 \cos 3 \delta \rangle = \sum_{r,s=1,2} \langle 0^+_1 \| \hat{Q} \| 2^+_r \rangle \langle 2^+_r \| \hat{Q} \| 2^+_s \rangle \langle 2^+_s \| \hat{Q} \| 0^+_1 \rangle . \]


Nuclear shapes studied by Coulomb excitation
Go forth and explore!

Approximately 3000 known (ca. 2013)
~7000 can exist in the laboratory

Pure samples, with specified Z and N, even at the single-atom level, can be isolated for study in the laboratory.

R&W Fig. 1.2