THE FLORIDA STATE UNIVERSITY

National Superconducting Cyclotron Facility

NSCL

Nuclear Reactions
Part III
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EBSS2013 July, 2013

Outline

- Introduction.
  - Resonances in atomic nuclei
  - Role of resonances in era of exotic beams
  - Relating observables to nuclear structure. R-matrix
- Resonance reactions with exotic beams. Experimental approaches
- Elastic and inelastic scattering with exotic nuclei. Nucleon Transfer reactions.

R-matrix theory

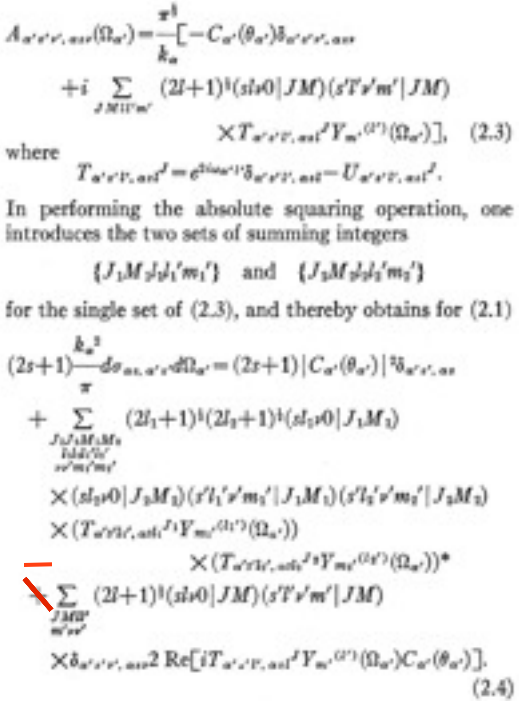
Multi-level, multi channel problem for charged particles with non-zero spin.

$$R \rightarrow R_{\alpha s\ell,\alpha' s'\ell'} = \sum_{\lambda} \frac{\gamma_{\lambda c}\gamma_{\lambda c'}}{E_\lambda - E}$$

$$U \rightarrow U_{\alpha s\ell,\alpha' s'\ell'}$$

$$\sigma_{\alpha\alpha} \sim C^2 + N^2 + C * N$$

$$\sigma_{\alpha\alpha'} \sim N^2$$



$$A_{\alpha's'\nu',\alpha s\nu}(\Omega_{\alpha'}) = \frac{\pi^{\frac{1}{2}}}{k_\alpha}\Big[-C_{\alpha'}(\theta_{\alpha'})\delta_{\alpha's'\nu',\alpha s\nu} + i \sum_{JMll'm'} (2l+1)^{\frac{1}{2}}(sl\nu 0|JM)(s'l'\nu'm'|JM) \times T_{\alpha's'l',\alpha sl}{}^{J} Y_{m'}{}^{(l')}(\Omega_{\alpha'})\Big], \quad (2.3)$$

where

$$T_{\alpha's'l',\alpha sl}{}^{J} = e^{2i\omega_{\alpha'l}}\delta_{\alpha's'l',\alpha sl} - U_{\alpha's'l',\alpha sl}{}^{J}.$$

In performing the absolute squaring operation, one introduces the two sets of summing integers

$$\{J_1M_1l_1l_1'm_1'\} \quad \text{and} \quad \{J_2M_2l_2l_2'm_2'\}$$

for the single set of (2.3), and thereby obtains for (2.1)

$$(2s+1)\frac{k_\alpha^2}{\pi} d\sigma_{\alpha s,\alpha's'} d\Omega_{\alpha'} = (2s+1)|C_{\alpha'}(\theta_{\alpha'})|^2\delta_{\alpha's',\alpha s}$$

$$+ \sum_{\substack{J_1J_2M_1M_2\\ l_1l_2l_1'l_2'\\ \nu\nu'm_1'm_2'}} (2l_1+1)^{\frac{1}{2}}(2l_2+1)^{\frac{1}{2}}(sl_1\nu 0|J_1M_1)$$

$$\times (sl_2\nu 0|J_2M_2)(s'l_1'\nu'm_1'|J_1M_1)(s'l_2'\nu'm_2'|J_2M_2)$$

$$\times (T_{\alpha's'l_1',\alpha sl_1}{}^{J_1} Y_{m_1'}{}^{(l_1')}(\Omega_{\alpha'}))$$

$$\times (T_{\alpha's'l_2',\alpha sl_2}{}^{J_2} Y_{m_2'}{}^{(l_2')}(\Omega_{\alpha'}))^*$$

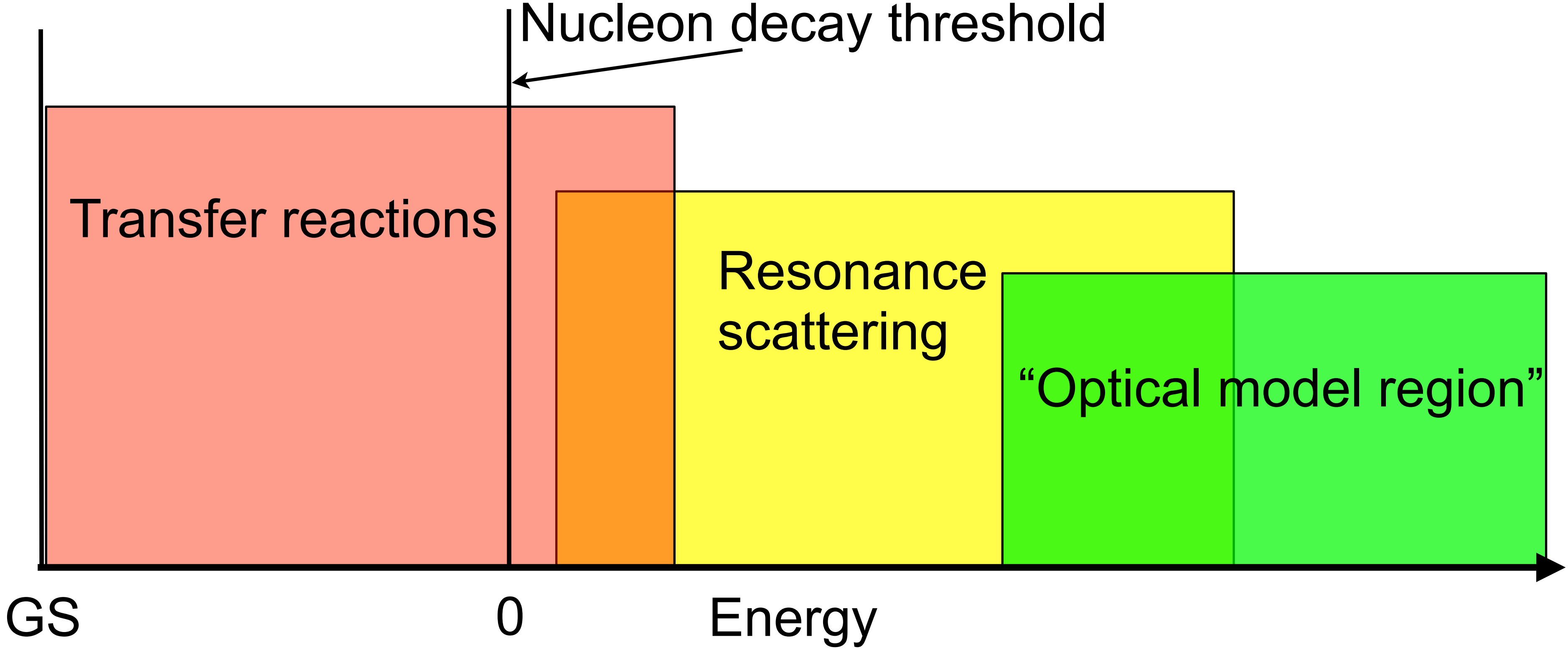
$$\overset{-}{\cancel{+}} \sum_{\substack{JMll'\\ m'\nu\nu'}} (2l+1)^{\frac{1}{2}}(sl\nu 0|JM)(s'l'\nu'm'|JM)$$

$$\times \delta_{\alpha's'\nu',\alpha s\nu} 2\,\mathrm{Re}[iT_{\alpha's'l',\alpha sl}{}^{J} Y_{m'}{}^{(l')}(\Omega_{\alpha'})C_{\alpha'}(\theta_{\alpha'})]. \quad (2.4)$$

Available codes: SAMMY (Oak Ridge)
AZURE (Notre Dame)
MinRmatrix (FSU)

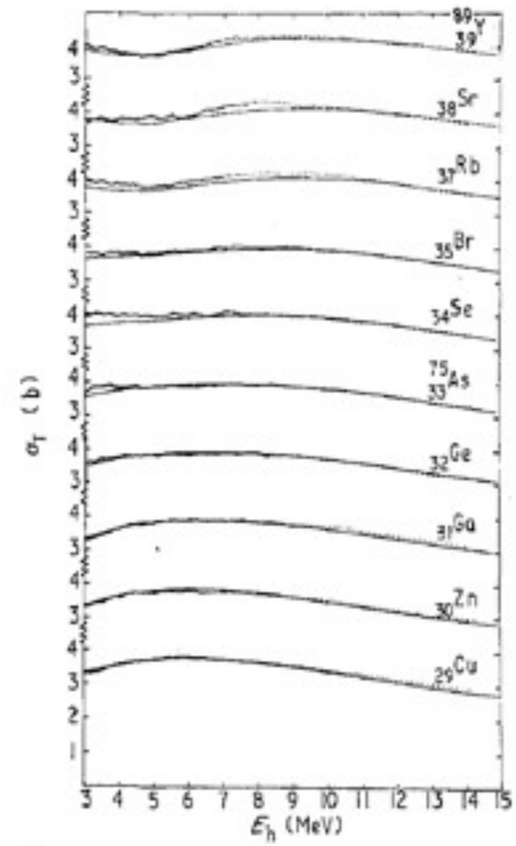
A.M. Lane and R.G. Thomas, Rev. of Mod. Phys., 30 (1958) 257

Elastic and Inelastic scattering. Transfer Reactions



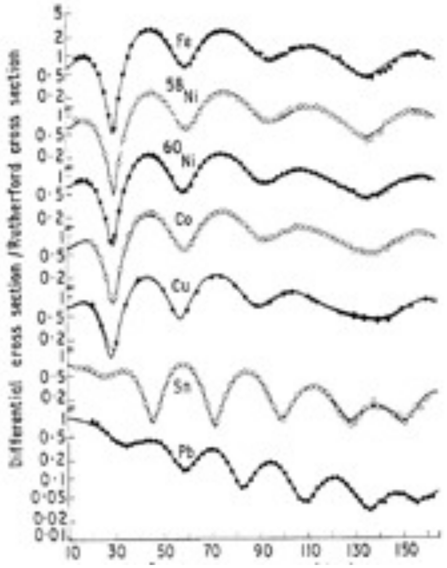
Elastic and inelastic scattering

Neutron total cross sections



$$\nabla^2\Psi + \frac{2\mu}{\hbar^2}(E - V)\Psi = 0$$

$$V(r) = V_C(r) + Vf_1(r) + i(W_v f_2(r) + W_s g(r)) + \left(\frac{\hbar}{m_\pi c}\right)^2 \frac{V_S}{r}\frac{df_3(r)}{dr} L.\sigma$$



Differential cross sections for proton elastic scattering

Elastic and inelastic scattering

Double Folding:

$$V_{\mathrm{bare}}(r) = \int\int \rho_{\mathrm{p}}(\vec{r_{\mathrm{p}}})\rho_{\mathrm{t}}(\vec{r_{\mathrm{t}}})v(\vec{r_{\mathrm{tp}}})\mathrm{d}\vec{r_{\mathrm{p}}}\mathrm{d}\vec{r_{\mathrm{t}}},$$

DWBA:

$$M^{(\mathrm{post})}(\mathbf{k}_{pF}, \mathbf{k}_{dA}) = \langle \Phi_f^{(-)} | \Delta V_{pF} | \Psi_i^{(+)} \rangle,$$

$$\sigma \sim |M|^2$$

$$\tilde{M}^{(\mathrm{post})}(\mathbf{k}_{pF}, \mathbf{k}_{dA}) = \langle \Phi_f^{(-)} | \Delta V_{pF} | \Phi_i^{(+)} \rangle.$$

$$\Phi_f^{(-)} = \chi_{pF}^{(-)} \varphi_F$$

$$\Phi_i^{(+)} = \varphi_d \varphi_A \chi_{dA}^{(+)}$$

Coupled Channels:

$$[E_i - H_i]\,\psi_i(\mathbf{R}_i) = \sum_{j \neq i} \langle \phi_i | \mathcal{H} - E | \phi_j \rangle\, \psi_j(\mathbf{R}_j).$$

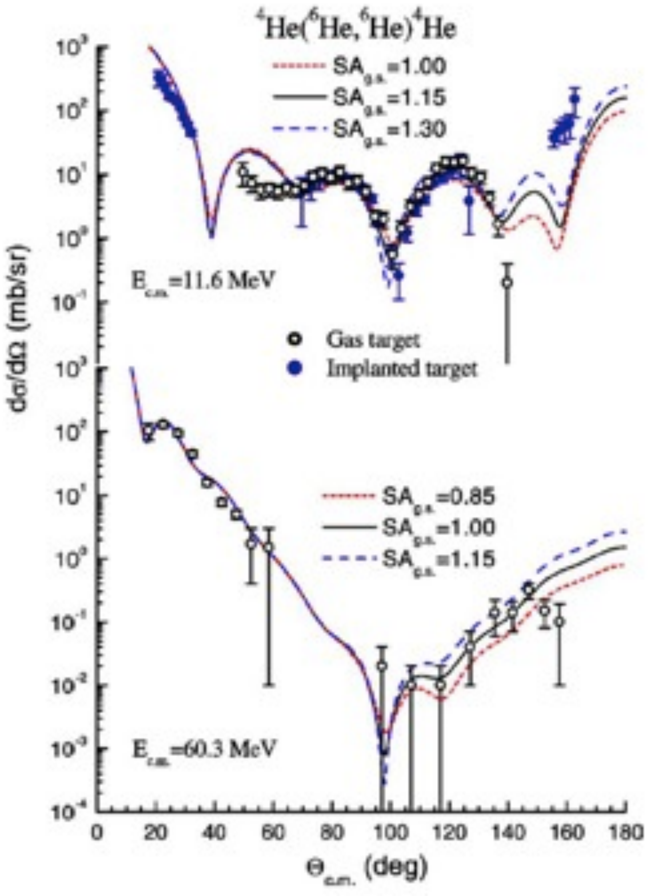
Continuum Discretised Coupled Channels:

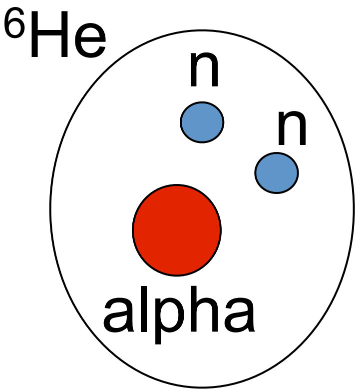
$$\Phi(r) = \sqrt{\frac{2}{\pi N}} \int_{k_1}^{k_2} w(k)\phi_k(r)\,\mathrm{d}k,$$

$$\text{where } N = \int_{k_1}^{k_2} |w(k)|^2\,\mathrm{d}k,$$

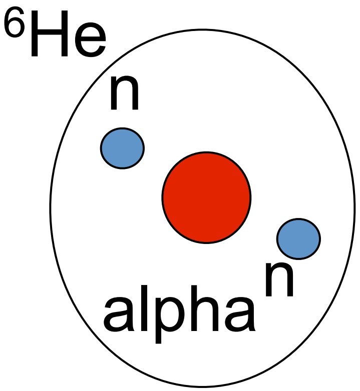
Review paper: N. Keeley, at. el., Prog Part. and Nucl. Phys. 63 (2009) 396.

Elastic scattering. $^4He(^6He,^6He)$





di-neutron configuration

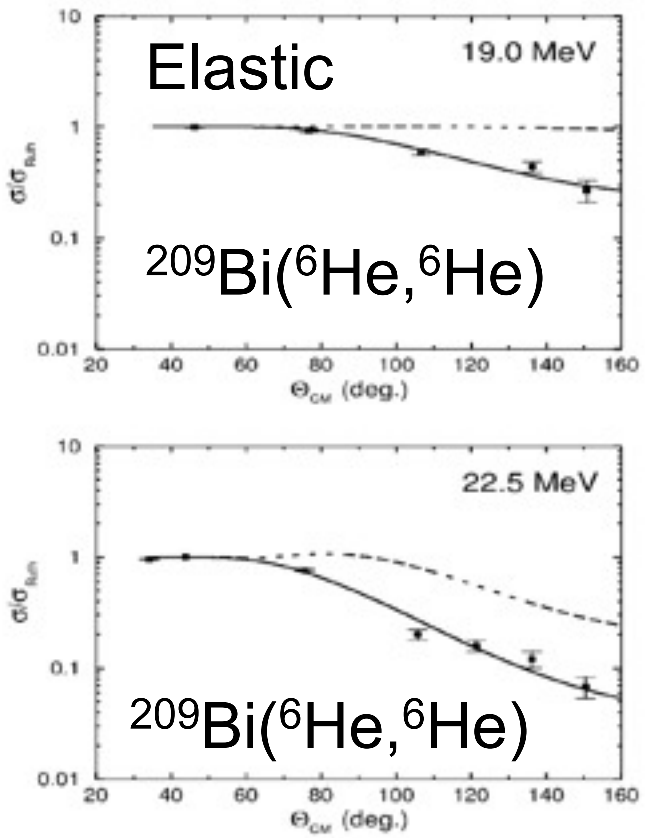


cigar-like

G.M. Ter-Akopian, Phys. Lett. B 426 (1998) 251
R. Raabe, Phys. Lett. B 458 (1999) 1
D.T. Khoa, W. von Oertzen Phys. Lett. B 595 (2004)

Coupled Reaction Channel calculations indicate strong 2n di-neutron configuration for the ^6He g.s.

^{6}He+^{209}Bi



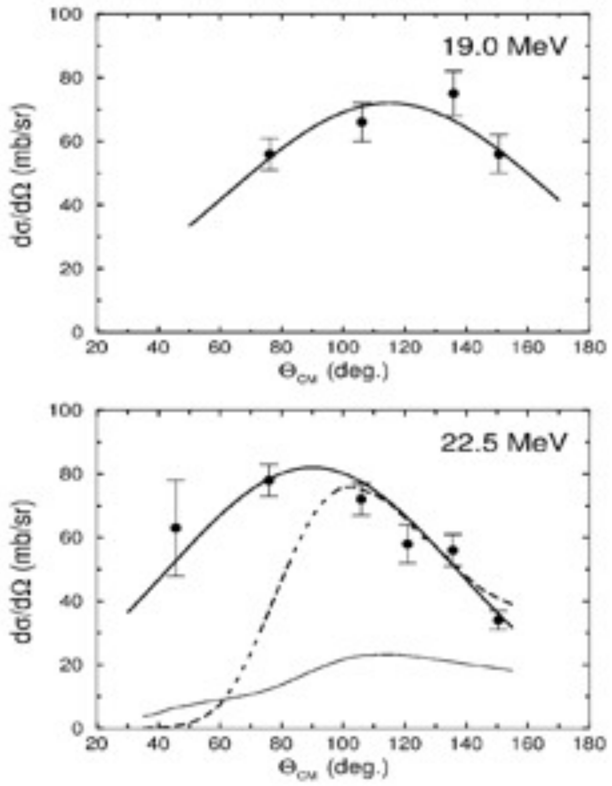
Elastic

^{209}Bi(^{6}He,^{6}He)

^{209}Bi(^{6}He,^{6}He)

$$\sigma_{el} \sim (1 - S)^2$$
$$\sigma_r \sim (1 - |S|^2)$$

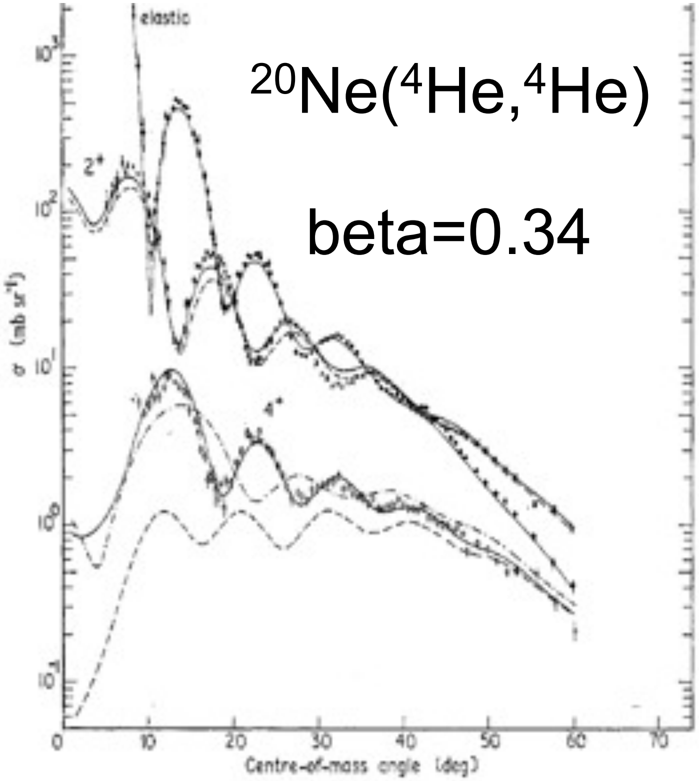
It was found that reaction cross section is strongly enhance, indicating that neutron wave function is radially extended



^{209}Bi(^{6}He,alpha)

E.F. Aguilera, et al., Phys. Rev. Lett. 84 (2000)

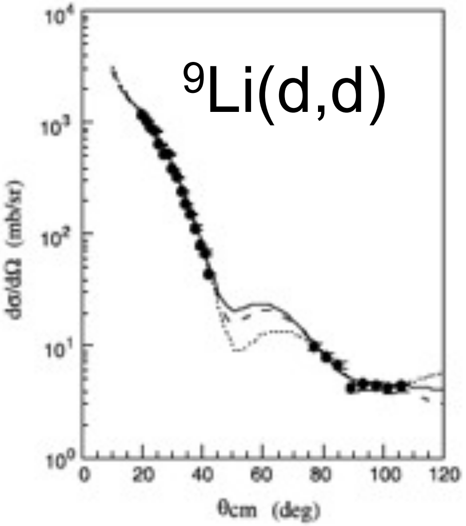
Inelastic scattering



^{20}Ne(^{4}He,^{4}He)

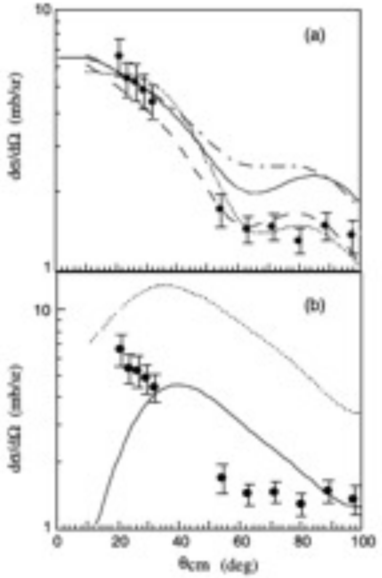
beta=0.34

P.E. Hodgson, Rep. Prog. Phys. 34 (1971) 765



^{9}Li(d,d)

^{9}Li(d,d')



beta=0.4 - quadrupole deformation parameter

H. Al Falou, et al., Phys. Lett. B 721 (2013) 224

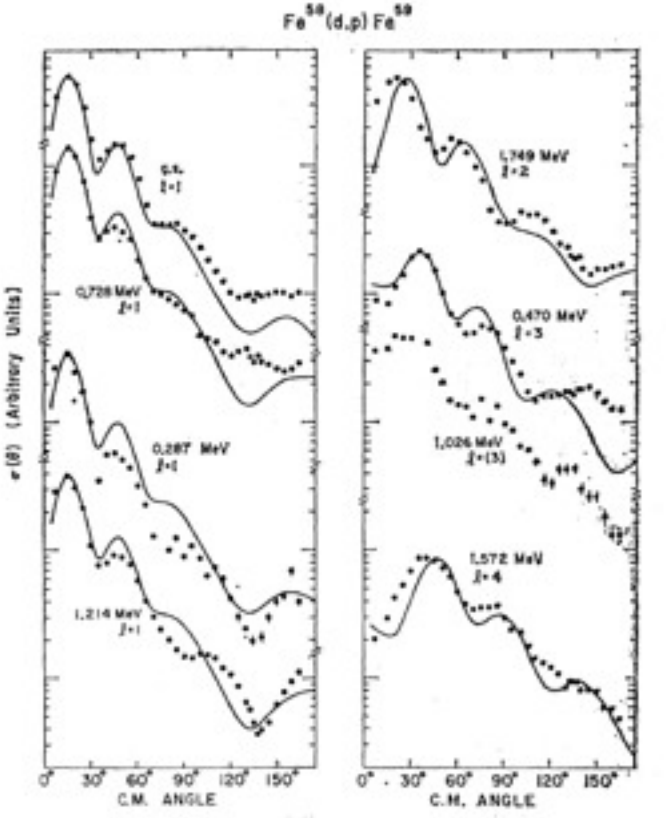
Transfer reactions

Bound and unbound but very narrow (near threshold states) can be populated and studied using nucleon transfer reactions.

Simple nucleon transfer reactions with rare isotope beams at energies ~10 MeV/u or below are great and now popular tools to study structure of exotic nuclei.

The most useful reactions are (d,p); (t,p); (d,^{3}He); (^{3}He,d); (p,d); (p,t);

Angular distribution for the transfer reaction is determined by the transferred angular momentum - therefore it is easy to determine the l-value.



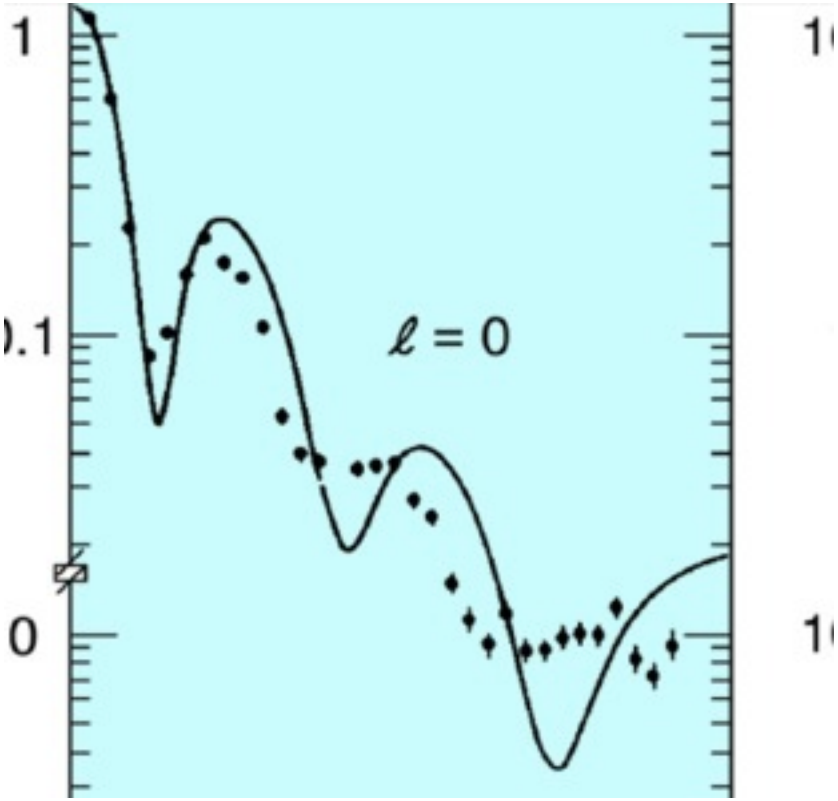
Klema, Lee, Schiffer Phys. Rev. 161 (1967)

$$q = |k_f - k_i| \approx 2k\sin(\theta/2) \sim \frac{\ell}{R}$$

With l-value know the spin-parity can be determined (often not uniquely)

States with what spin-parities can be populated in L=2 transfer in ^{58}Fe(d,p) reaction?

At what angle the cross section maximum is expected for L=0 transfer?



$$q = |k_f - k_i| \approx 2k\sin(\theta/2) \sim \frac{\ell}{R}$$

States of what spin (low or high) are going to be favored in a reaction that has very negative Q-value?

(For example, ^{58}Fe(^{4}He,^{3}He); Q=-14 MeV)

Spectroscopic factors

The spectroscopic factor is the overlap between initial and the final state in a reaction channel. Cross section for the transfer reaction is proportional to the spectroscopic factor. For resonance state spectroscopic factor can be related to the reduced width.

C* -> a+A

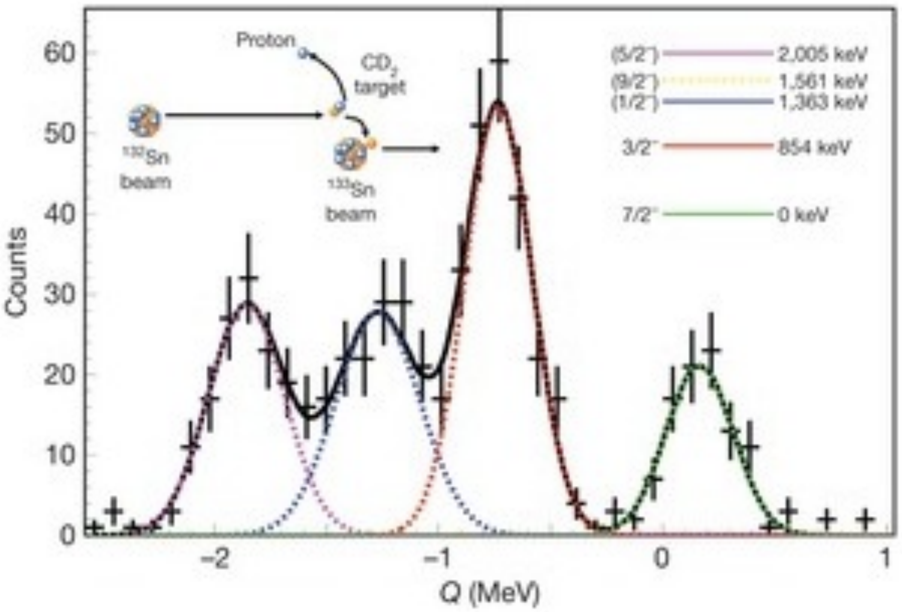
$$S \sim \int [\phi(a) \times \phi(A)] \phi(C^*) ds$$

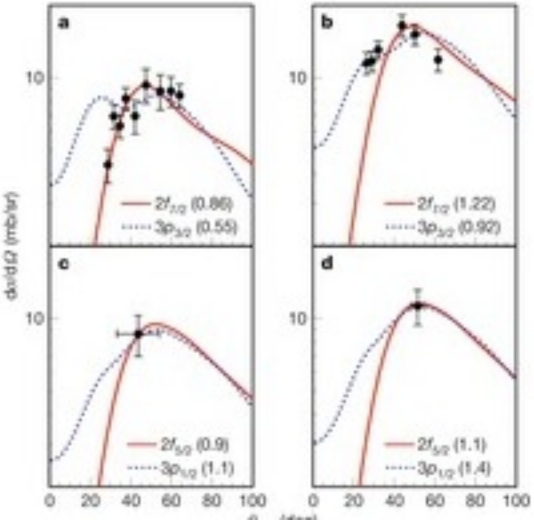
$$S = \frac{\gamma^2}{\gamma_{sp}^2}$$

ad + A -> aA + d

$$\frac{d\sigma}{d\Omega}_{\exp} = S_{\alpha d} S_{\alpha A} \frac{d\sigma}{d\Omega}_{DWBA}$$

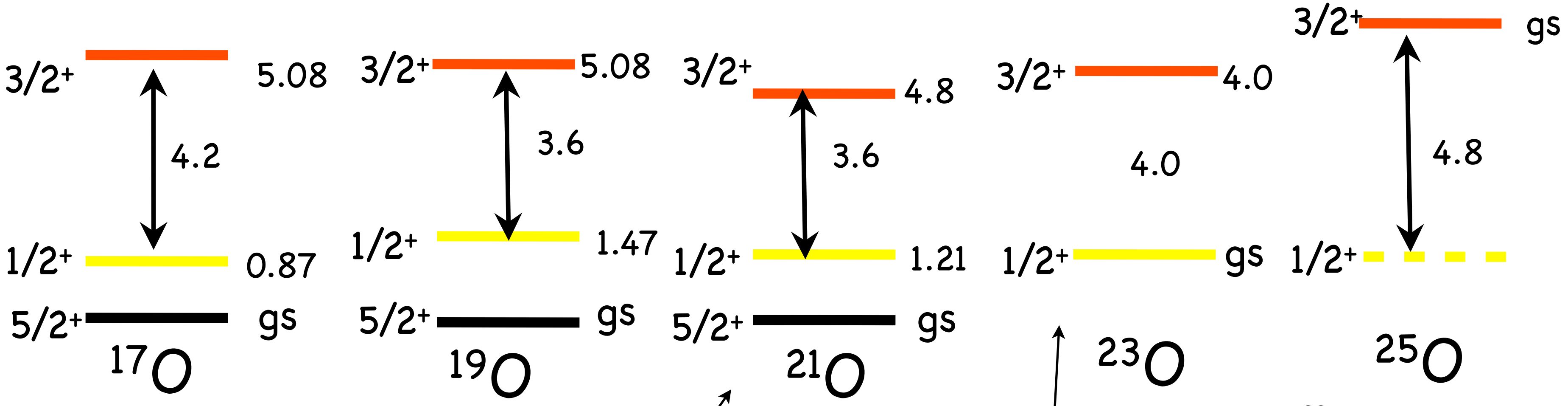
^{132}Sn(d,p)





K. Jones, et al., *Nature* **465**, 454–457 (27 May 2010)

Emergence of N=16 shell gap



$3/2^+$ 5.08
4.2
$1/2^+$ 0.87
$5/2^+$ gs
^{17}O

$3/2^+$ 5.08
3.6
$1/2^+$ 1.47
$5/2^+$ gs
^{19}O

$3/2^+$ 4.8
3.6
$1/2^+$ 1.21
$5/2^+$ gs
^{21}O

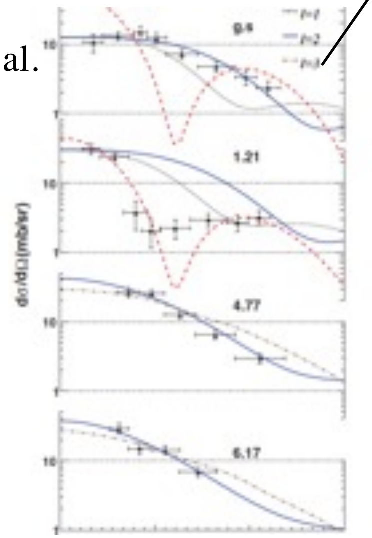
$3/2^+$ 4.0
4.0
$1/2^+$ gs
^{23}O

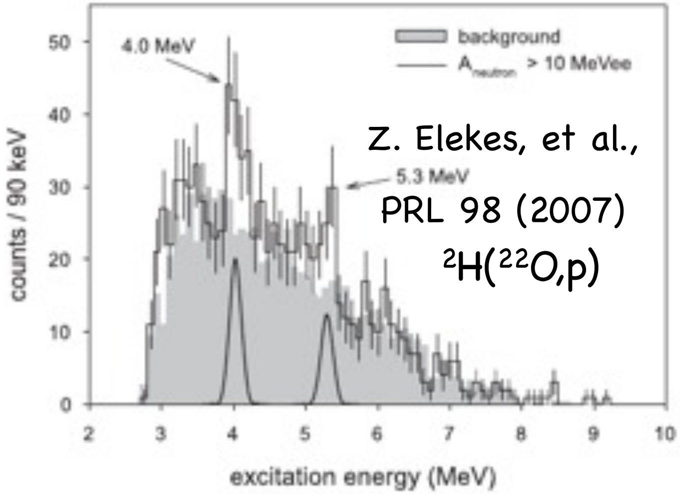
$3/2^+$ gs
4.8
$1/2^+$
^{25}O

C. Hoffman, et al., PRL 100 (2008)

B. Fernandez-Domınguez, et al. Phys. Rev. C 84, 011301(R) (2011)

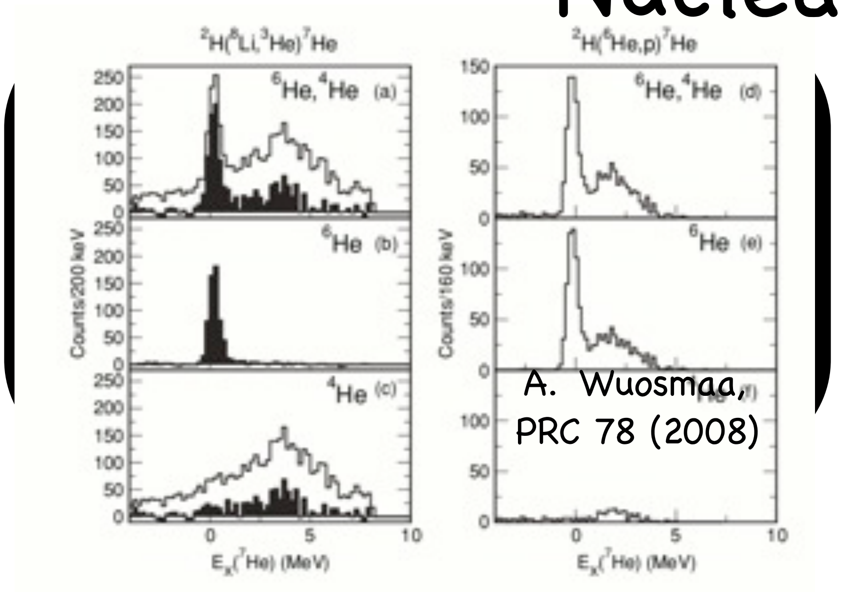
^{2}H(^{20}O,p)





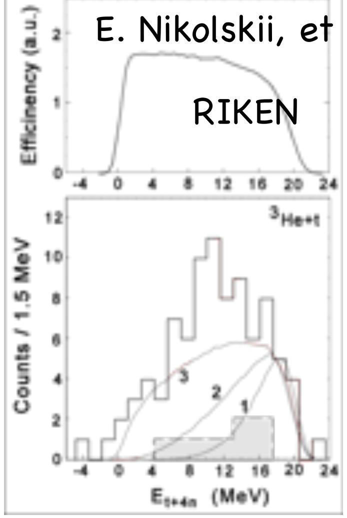
Z. Elekes, et al., PRL 98 (2007)
^{2}H(^{22}O,p)

Nuclear extremes



A. Wuosmaa, PRC 78 (2008)

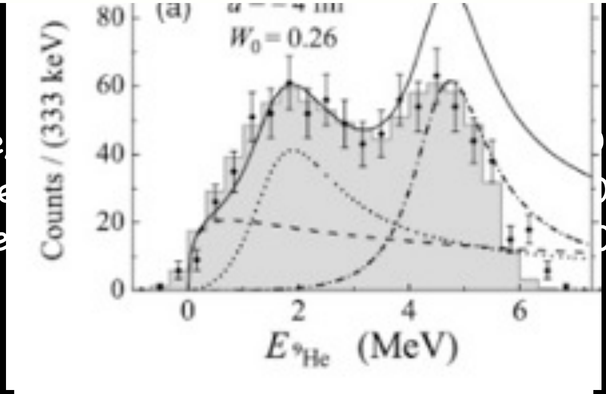
$^1H(^8He,2p)$ A. …), (2003)
$^2H(^8He,^3He)$ C …) (2007)
$^{12}C(^8He,^{13}N)$ … (2007)
E. Nik… L



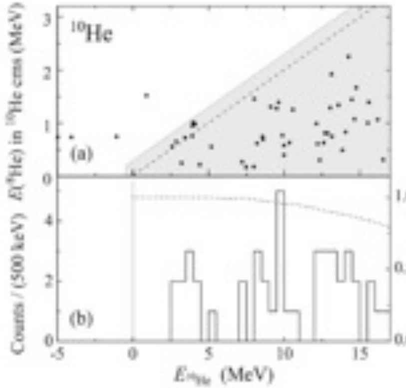
E. Nikolskii, et al.,

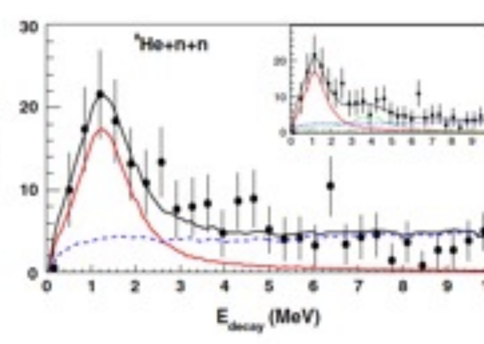
RIKEN

$^1H(^8He,$ … 03)
$^{12}C(^8He$ … 01)
$^2H(^8He$ … 007)



$^2H(^8He,p)$ M.S.Golovkov, et al., PRC 76 (2007)





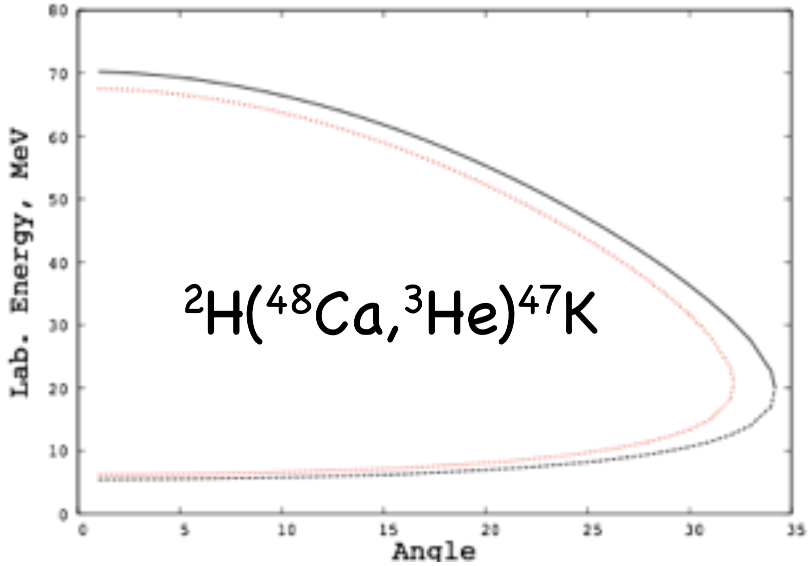
Were is the GS?

$^3H(^8He,p)^{10}He$ M.S. Golovkov, et al., PL B 672 (2009)

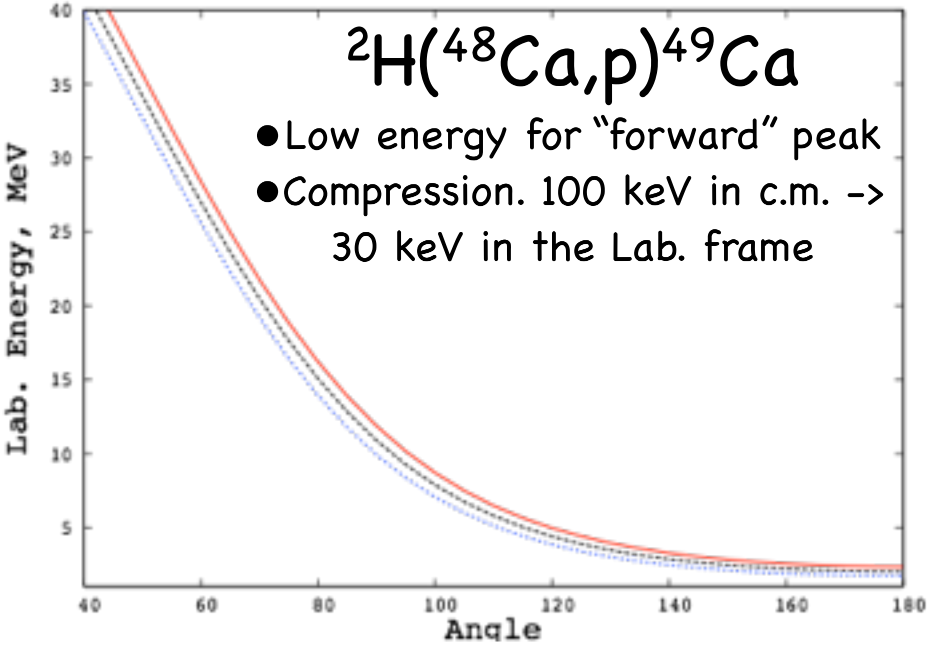
Inverse kinematics with transfer reactions

HI nucleon(s) drop off

HI nucleon(s) pick up



$^{2}H(^{48}Ca,^{3}He)^{47}K$



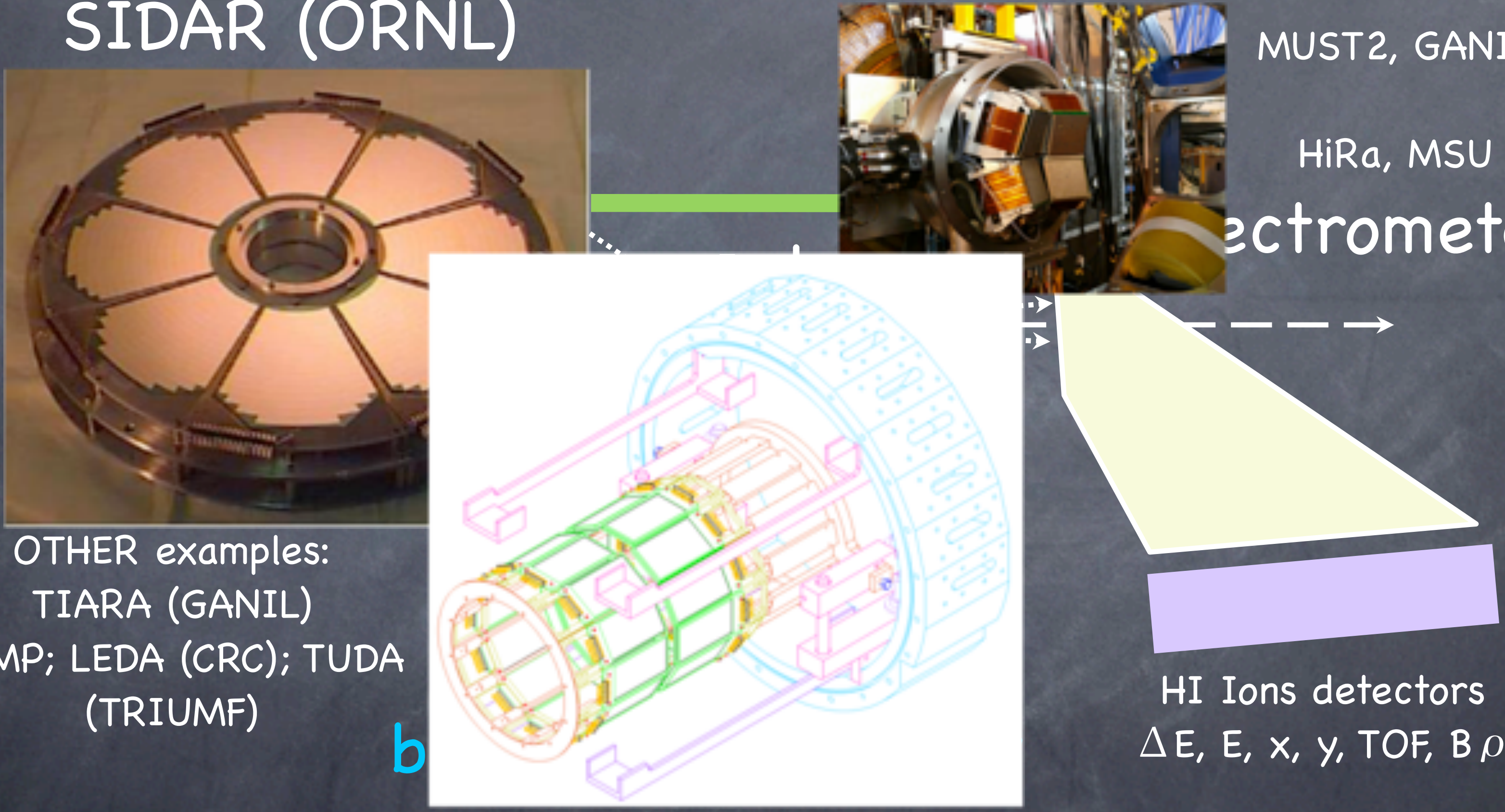
$^{2}H(^{48}Ca,p)^{49}Ca$

- Low energy for "forward" peak
- Compression. 100 keV in c.m. -> 30 keV in the Lab. frame

E_{beam} ~ 10 MeV/u

“Typical” experimental setup for transfer reactions experiment with

SIDAR (ORNL)



MUST2, GANIL

HiRa, MSU

ectrometer

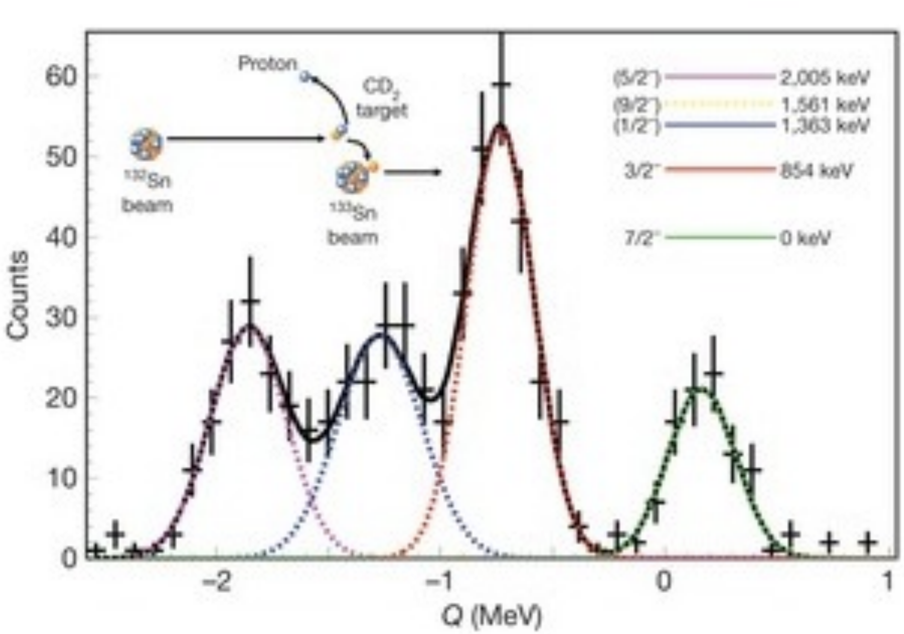
OTHER examples:
TIARA (GANIL)
LAMP; LEDA (CRC); TUDA
(TRIUMF)

b

HI Ions detectors
ΔE, E, x, y, TOF, Bρ

Oak Ridge Rutgers U
Barrel Array

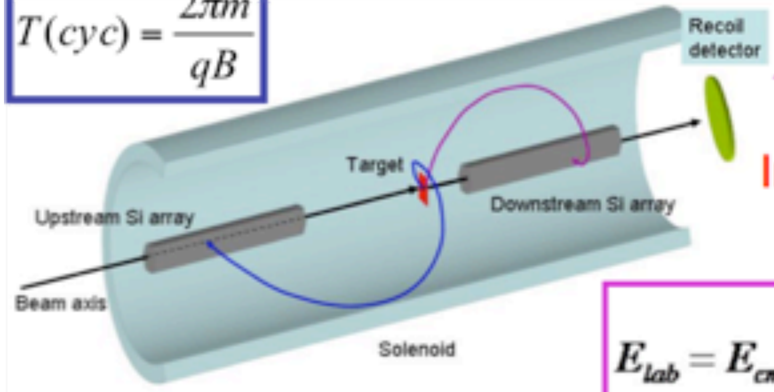
Typical energy resolution ~400 keV



HELIOS: Spectrometer for Inverse-kinematic reactions

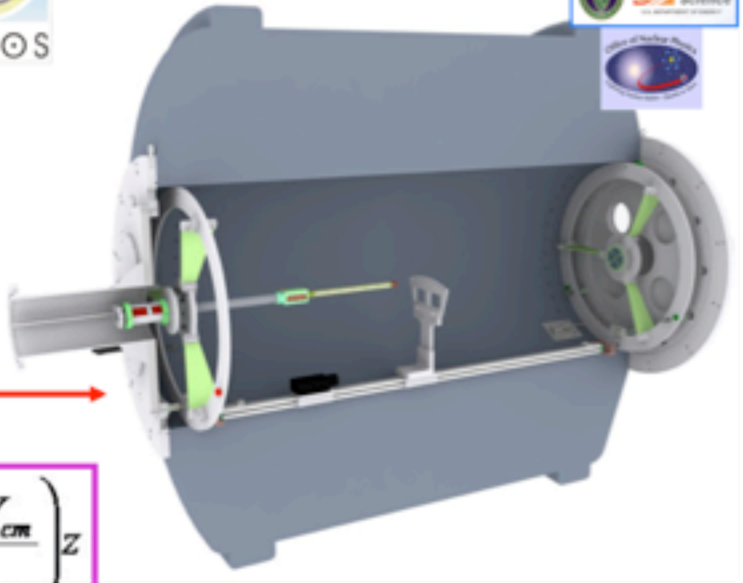
A. H. Wuosmaa et al, NIM A 580, 1290 (2007)

$$T(cyc) = \frac{2\pi m}{qB}$$



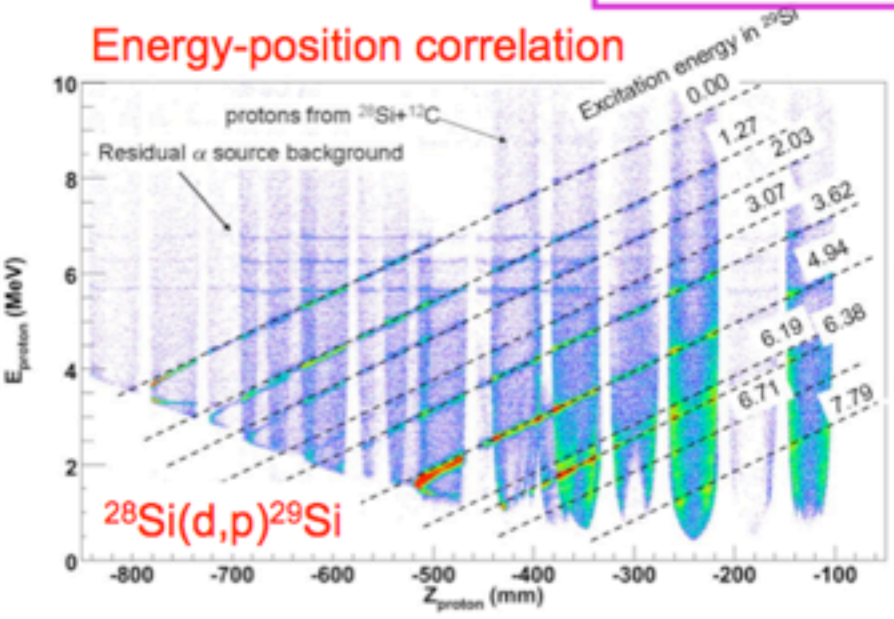
Concept ←

Implementation →

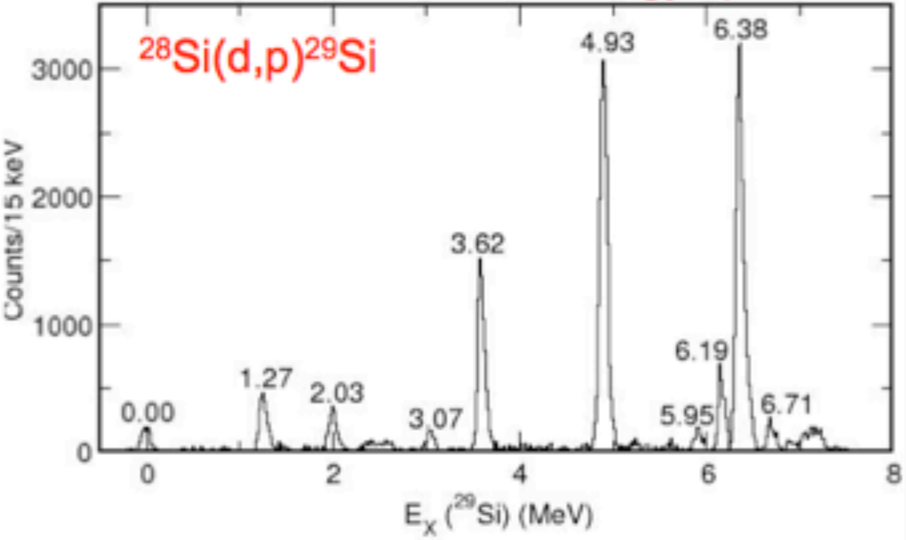


$$E_{lab} = E_{cm} - \frac{1}{2} m V_{cm}^2 + \left(\frac{m V_{cm}}{T_{cyc}}\right) z$$

Energy-position correlation



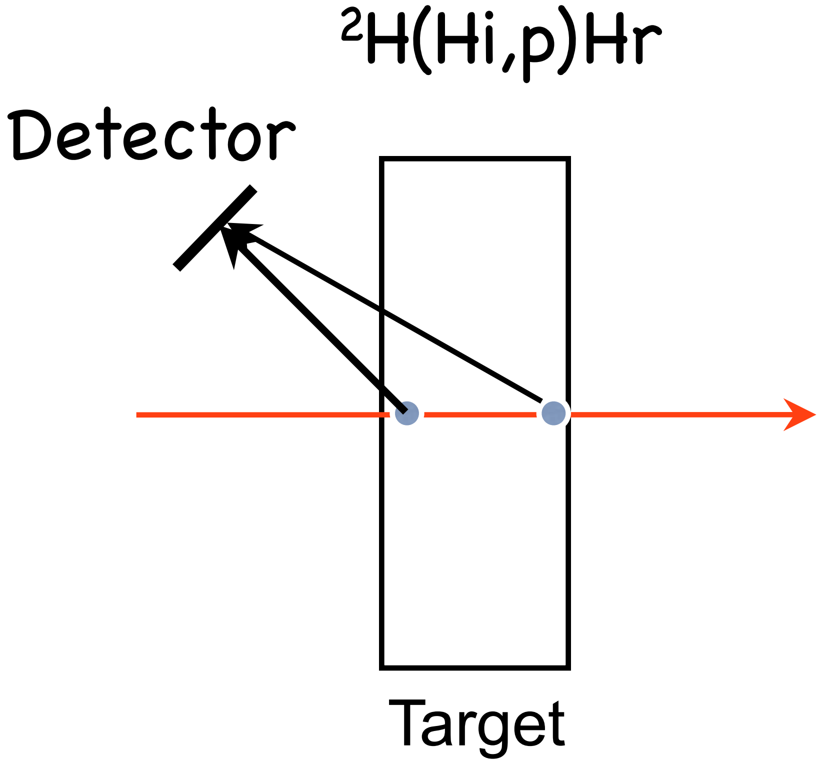
Excitation-energy spectrum



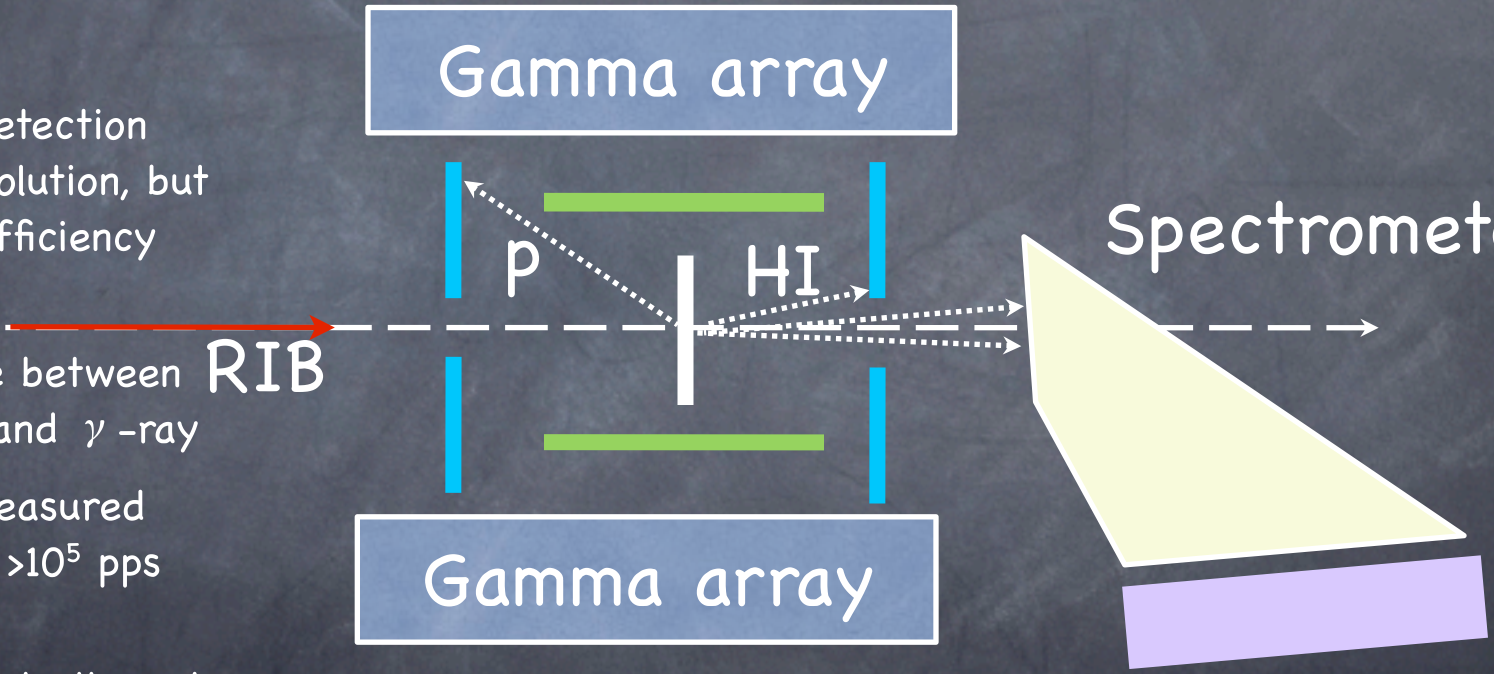
Slide is courtesy of A. Wuosmaa

Target thickness and resolution

- Target thickness is restricted by energy losses of light and heavy recoils
- To keep energy resolution <100 keV target should be <1 mg/cm^2
- With cross sections ≈1 mb beam intensity should be >10^3 pps



Si array + Gamma array + spectrometer



Gamma detection improves resolution, but reduces efficiency

Coincidence between light recoil and γ-ray can be measured with RIBs >10^5 pps

Thick target allows to perform measurements with beams >10^3 pps (no light recoil is measured)

d containing target

side Si array

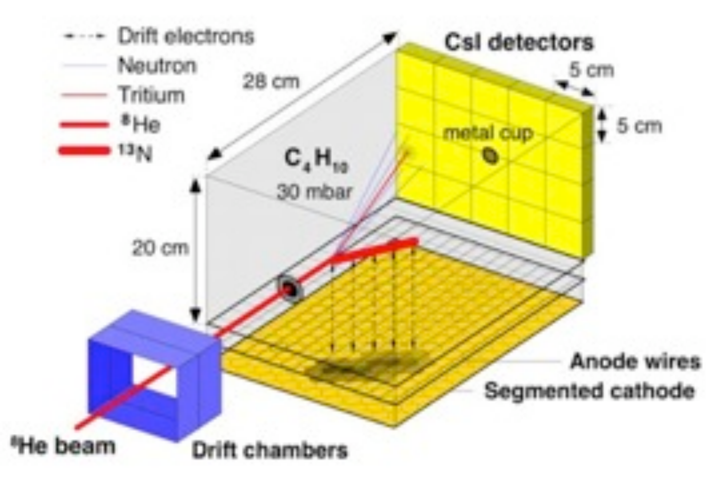
back/front Si array

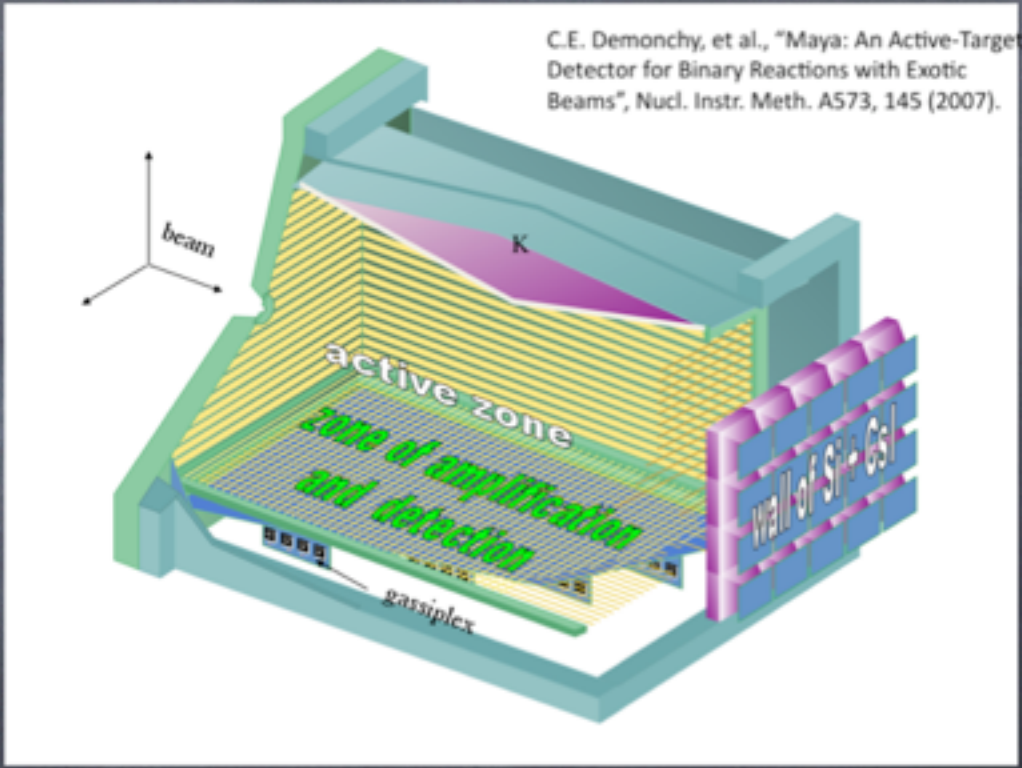
HI Ions detectors
ΔE, E, x, y, TOF, Bρ

Active Target

Transfer reactions with RIBs 10^3 pps and less require active target.

MAYA (Developed at GANIL, now at TRIUMF)

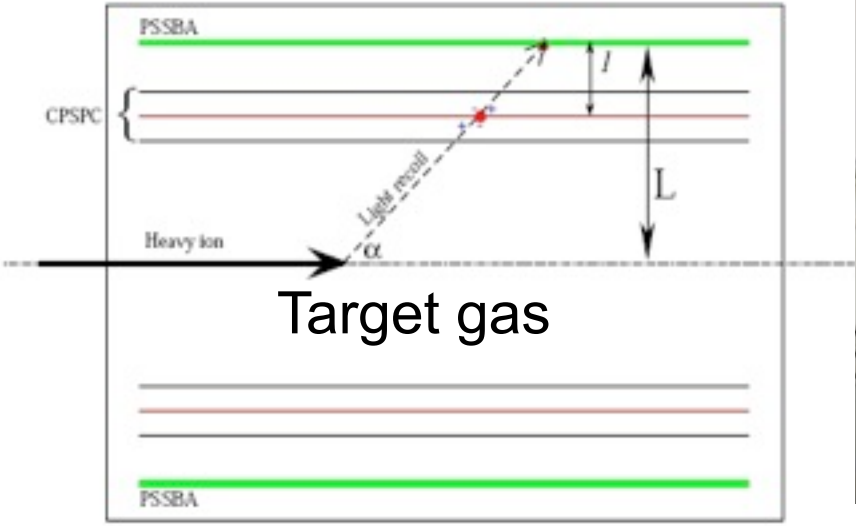



C.E. Demonchy, et al., "Maya: An Active-Target Detector for Binary Reactions with Exotic Beams", Nucl. Instr. Meth. A573, 145 (2007).

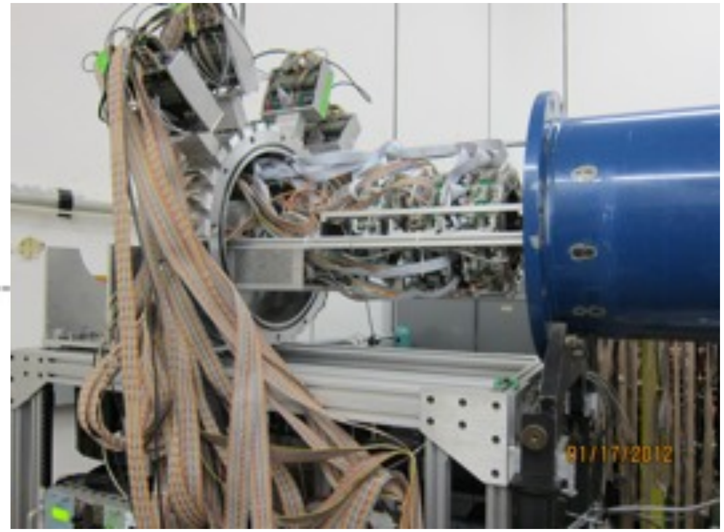
- AT-TPC – MSU
- TACTIC – TRIUMF
- ANASEN – LSU-FSU
- ACTAR – GANIL
- SAMURAI TPC - RIKEN

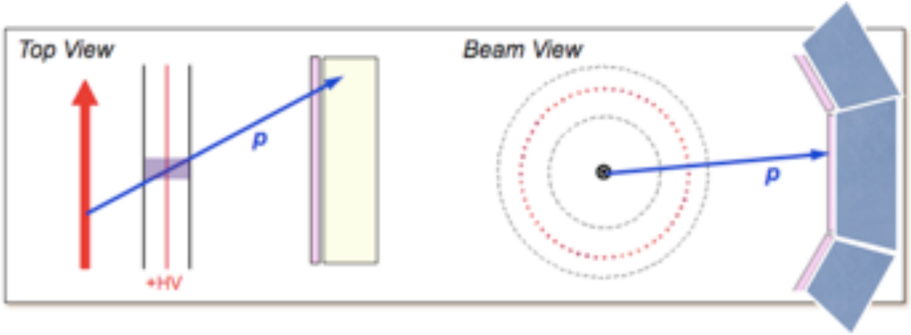
ANASEN

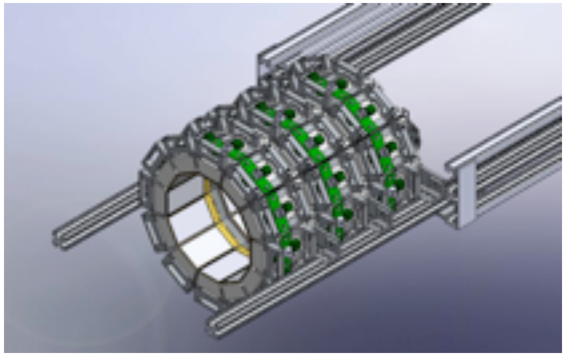
Array for Nuclear Astrophysics and Structure with Exotic Nuclei



Target gas

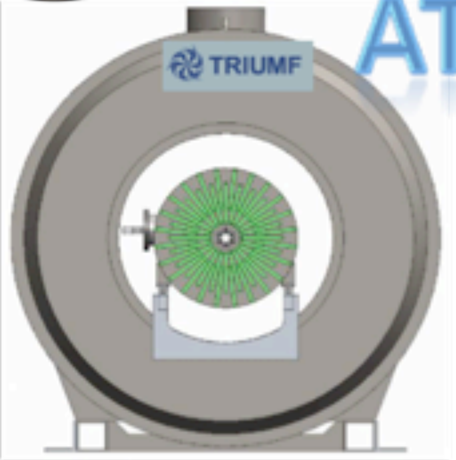


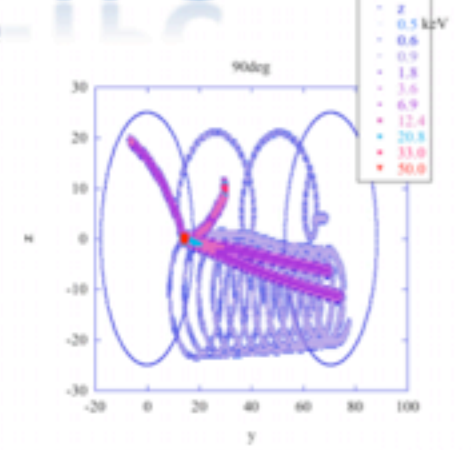


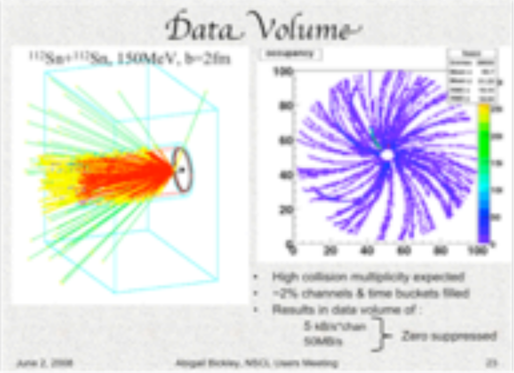


AT-TPC

W. Mittig, et al, MSU







Data Volume

$^{112}Sn+^{112}Sn$, 150MeV, b=2fm

- High collision multiplicity expected
- ~2% channels & time buckets filled
- Results in data volume of :

5 kB/s*chan
50MB/s
} Zero suppressed

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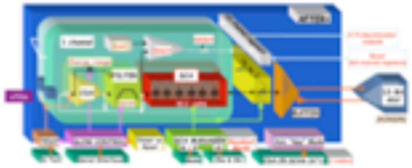
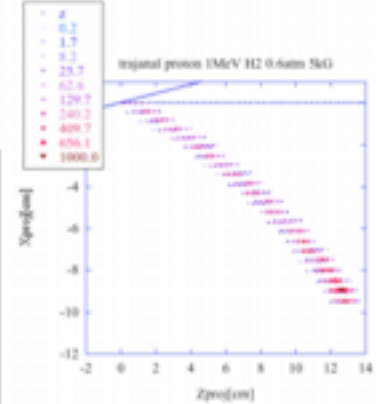
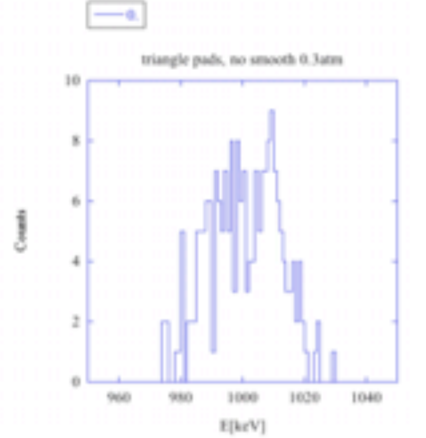
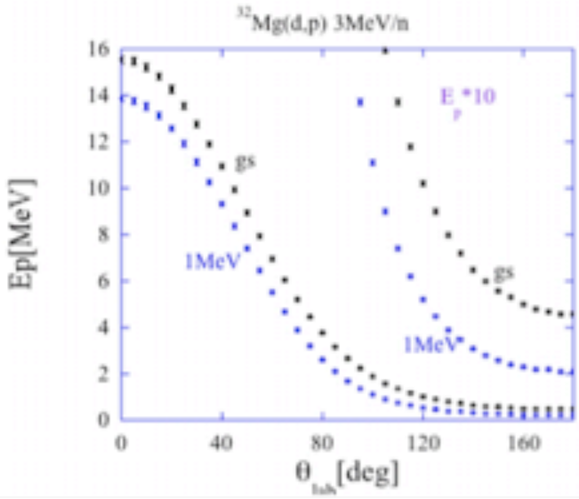
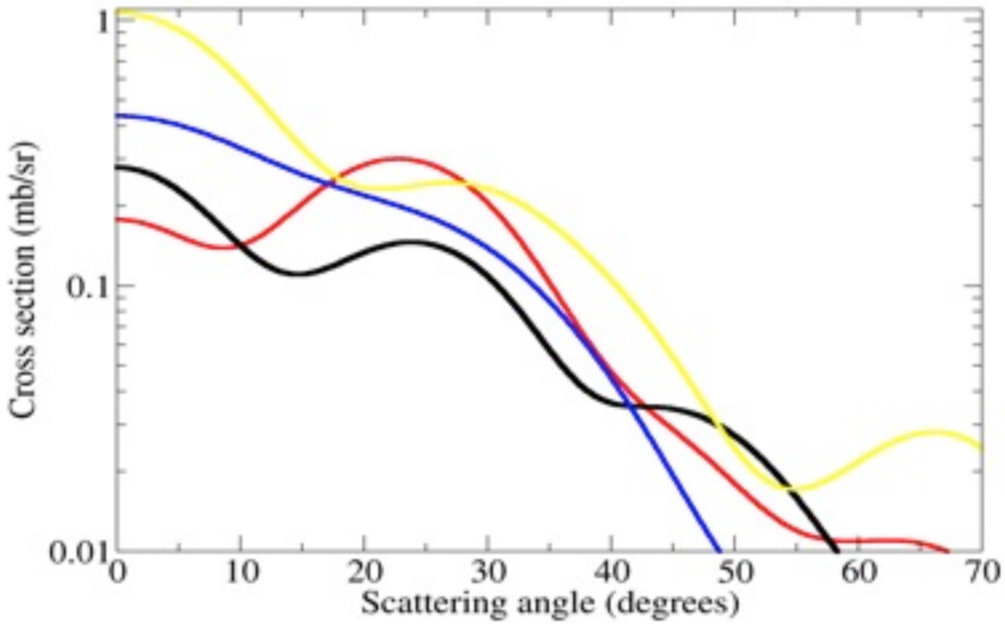








$^{13}C(^{6}Li,d)^{17}O(1/2^{+}$; 6.356 MeV) DWBA calculations at 60 MeV

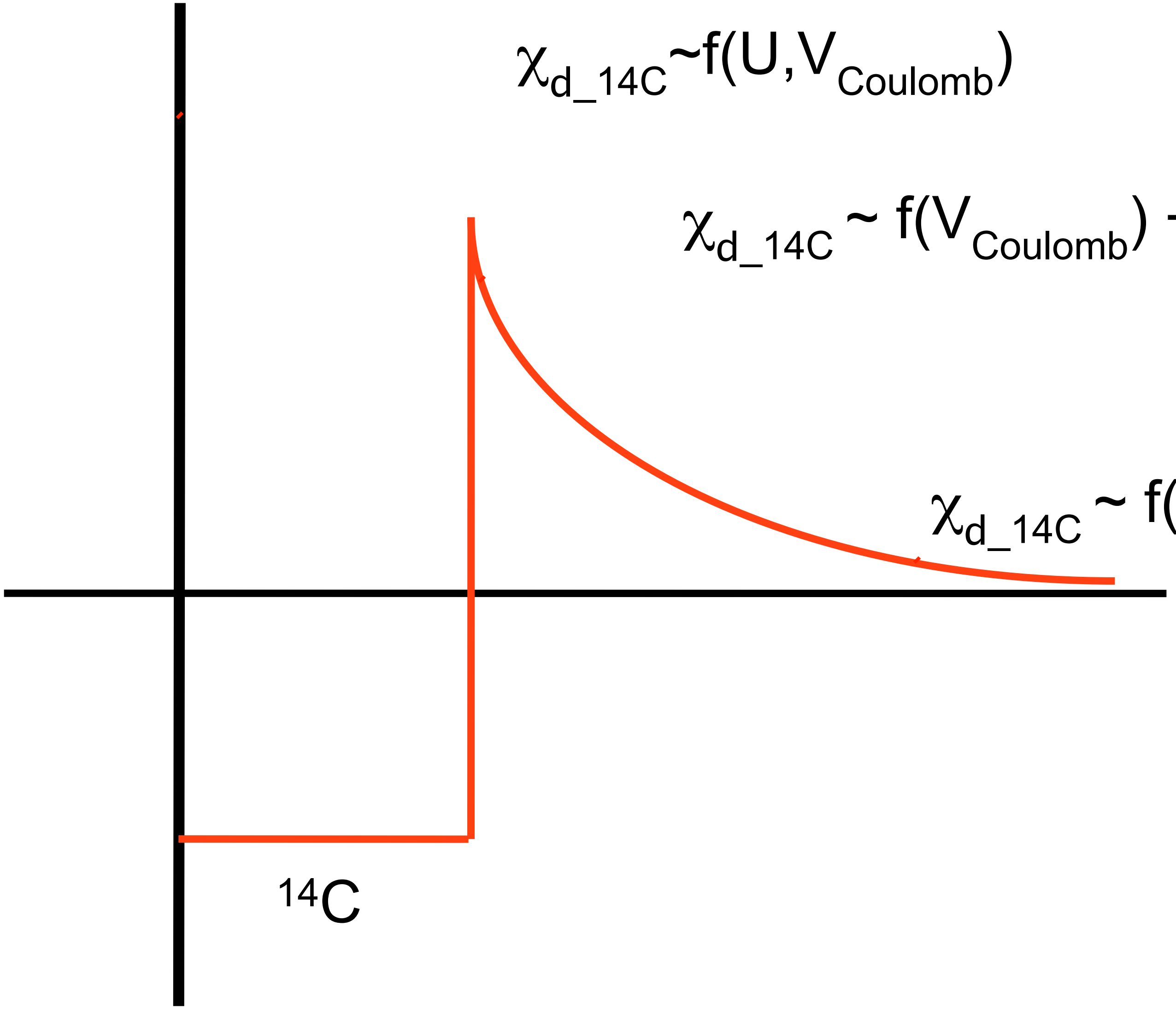


Black curve – optical potentials from S. Kubono, *et al.*, PRL 90 (2003) 062501;

Red curve – deuteron optical potential from T.K. Li, *et al., PRC 13 (1975) 55;* Blue curve – radius of the $^{13}C+\alpha$ formfactor decreased by 25%;

Yellow curve – +1 node in $^{13}C+\alpha$ wavefunction.

- ALL uncertainties can be drastically reduced if:
  - α transfer reaction is performed at sub-Coulomb energy. This eliminates dependence of the calculated cross section on optical potentials.
  - ANCs are extracted from experimental data. This eliminates dependence of the final result on the shape of form-factor binding potentials and number of wavefunction nodes.



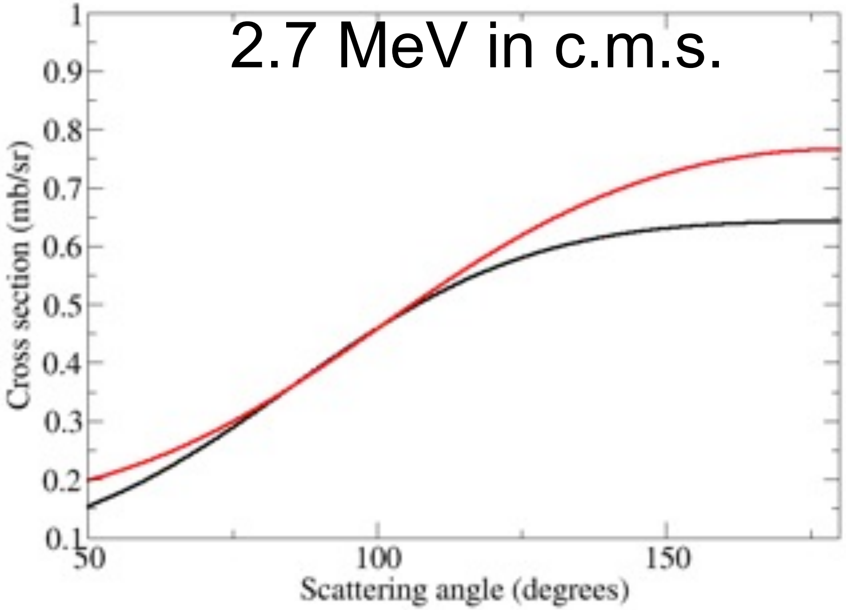
$\chi_{d\_14C} \sim f(U, V_{Coulomb})$

$\chi_{d\_14C} \sim f(V_{Coulomb}) + o(f(U))$

$\chi_{d\_14C} \sim f(V_{Coulomb})$

If reaction is performed at sub-Coulomb energy then variation of optical potential parameters produce only small variation in the DWBA cross section.

$^{13}C(^{6}Li,d)$



2.7 MeV in c.m.s.

ab+A -> aA + b

$$I_{ab} = \sqrt{S_{ab}}\varphi_{ab} = C_{ab}\frac{W}{r}$$

Model ab cluster wavefunction

$$\varphi_{ab} = b_{ab}\frac{W}{r}$$

Single-particle ab cluster wavefunction

$$C_{ab}^2 = S_{ab}b_{ab}^2$$

Definition of ANC through single-particle ANC

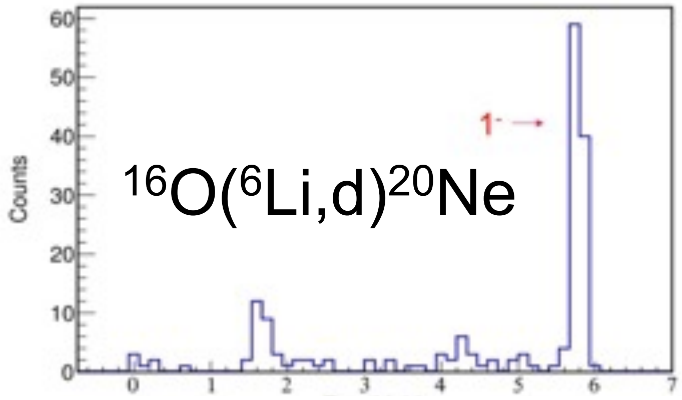
$$\frac{d\sigma}{d\Omega}_{\exp} = S_{\alpha d}S_{\alpha A}\frac{d\sigma}{d\Omega}_{DWBA}$$

$$\frac{d\sigma}{d\Omega}_{DWBA} \sim b_{\alpha d}^2 b_{\alpha A}^2 X$$

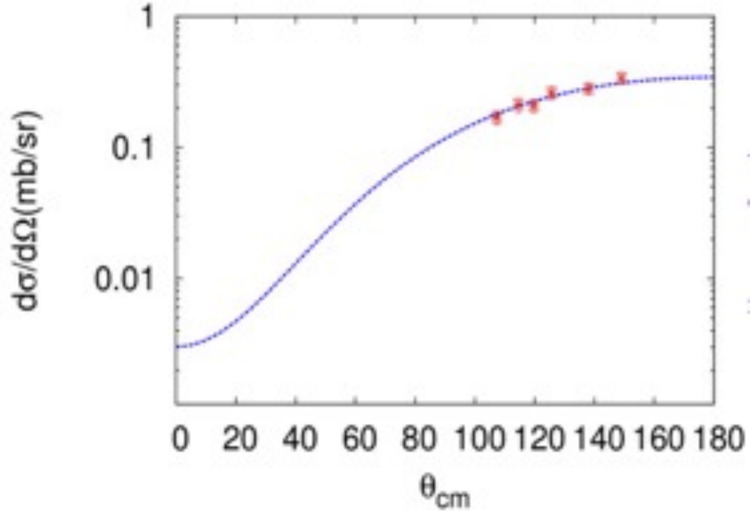
X depends only on entrance and exit channel optical potentials

$$C_{\alpha d}^2 C_{\alpha A}^2 \sim \frac{1}{b_{\alpha d}^2 b_{\alpha A}^2 X} b_{\alpha d}^2 b_{\alpha A}^2 \frac{d\sigma}{d\Omega}_{\exp}$$

The $^{13}C(a,n)$ reaction rate

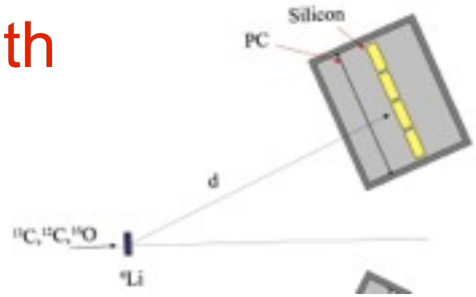


$^{16}O(^{6}Li,d)^{20}Ne$

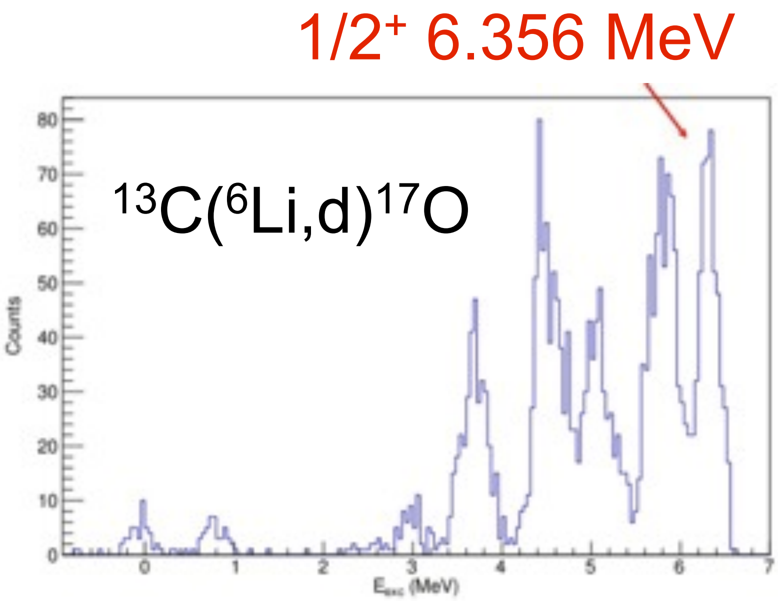


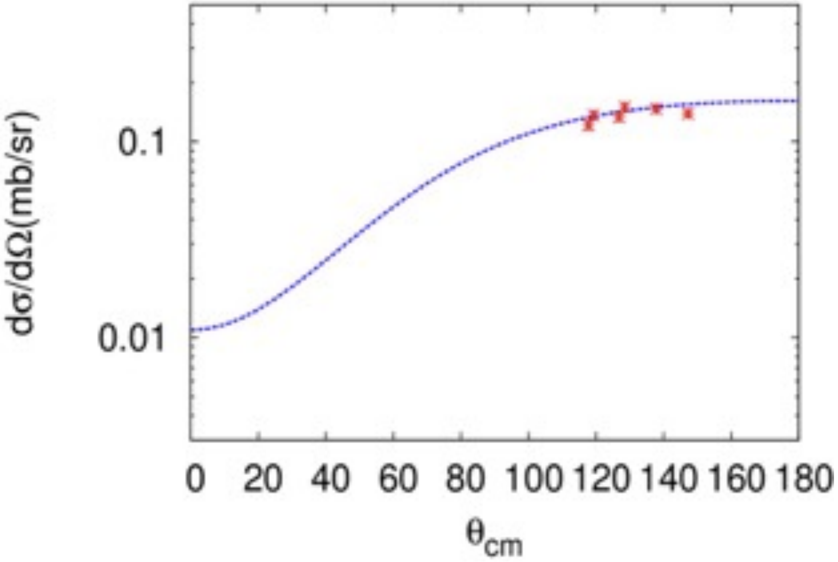
The known width of 1^- state at 5.9 MeV in ^{20}Ne is 28+/-2 eV

The partial alpha-width determined from the measured ANC is 29+/-3 eV



The ^{13}C(a,n) reaction rate





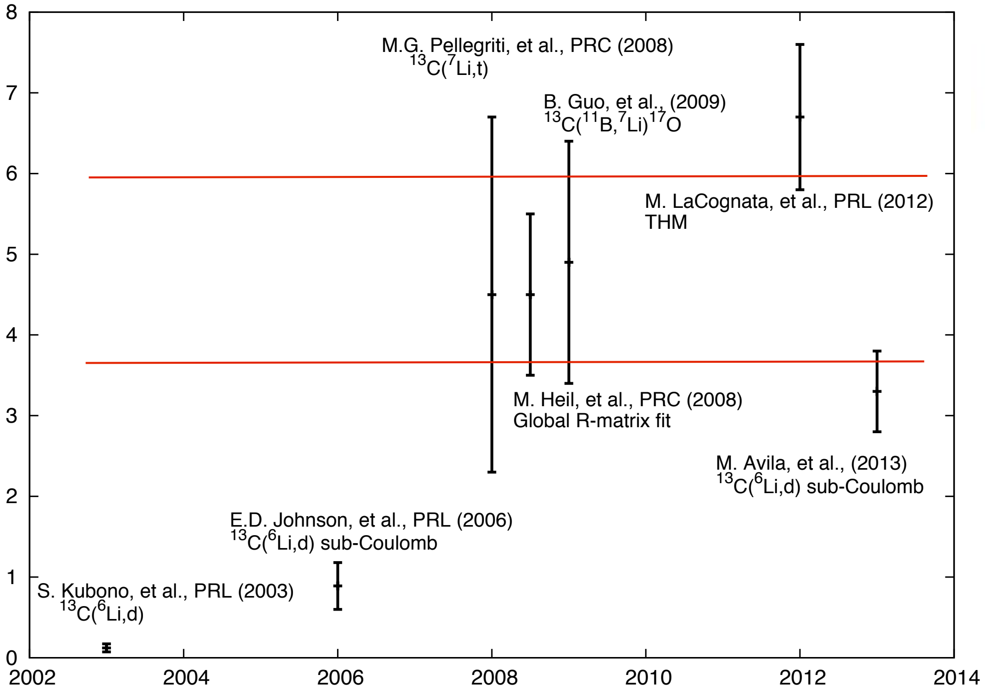
Coulomb modified ANC^2 for the 1/2$^+$ 6.356 MeV state is 3.3+/-0.5 fm^{-1}

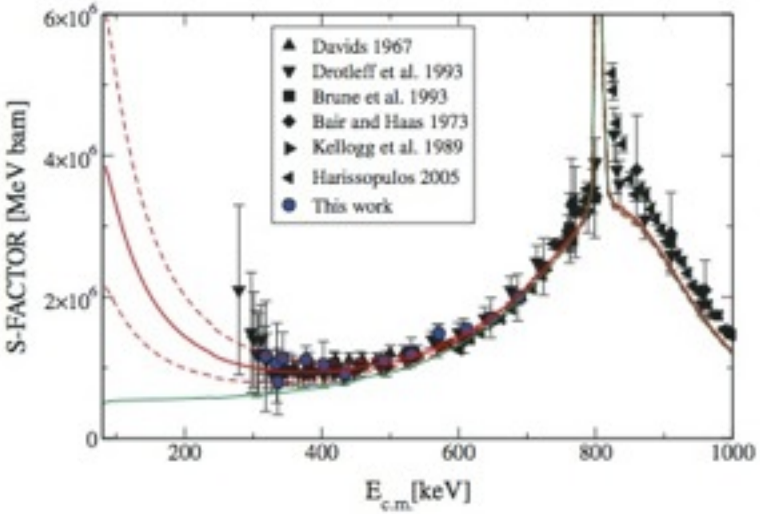
$$\gamma = \sqrt{\left(\frac{h^2}{2\mu R}\right)}\int \varphi_c^* \Psi_{^{18}O} d\xi$$

$$I = C\frac{W}{r}$$

$$\gamma^2 = \frac{h^2}{2\mu}\frac{W^2}{R}C^2$$

Coulomb modified ANC^2 (fm^{-1})





The 1/2$^+$ at 6.356 MeV in ^{17}O enhances the ^{13}C(alpha,n) cross section in Gamow window.

Survival probability of ^{13}C in the ^{13}C pocket is reduced due to enhanced rate. This limits the abundance of radioactive ^{60}Fe in the spectrum of AGB star.