THE FLORIDA STATE UNIVERSITY

National Superconducting Cyclotron Facility

Nuclear Reactions
Part I

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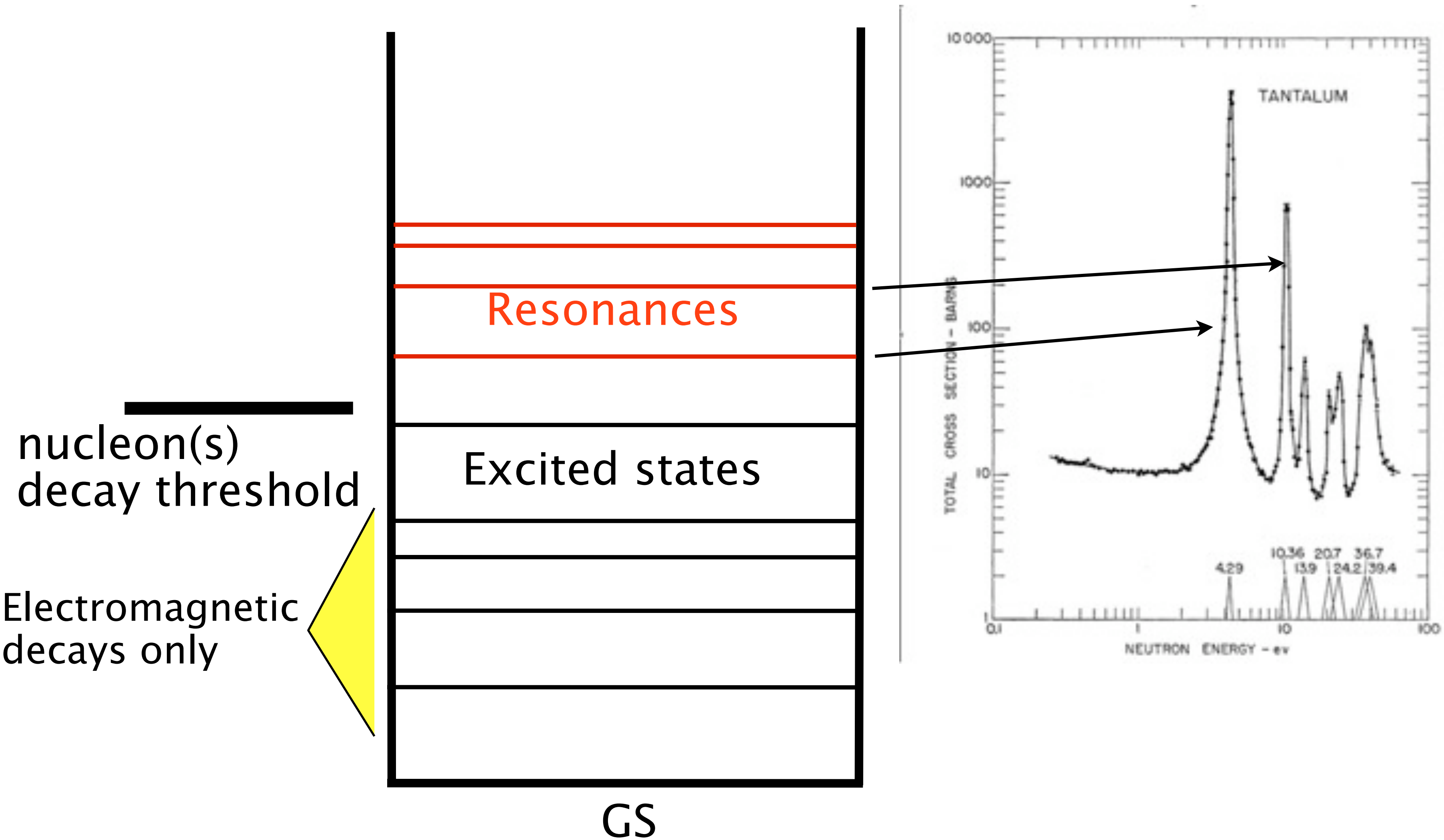
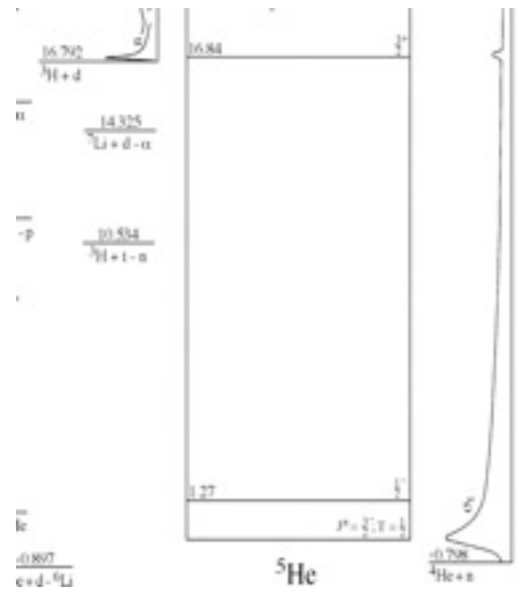
Outline

- Introduction.
  - Resonances in atomic nuclei
  - Role of resonances in era of exotic beams
  - Relating observables to nuclear structure. R-matrix
- Resonance reactions with exotic beams. Experimental approaches
- Elastic and inelastic scattering with exotic nuclei. Nucleon Transfer reactions.

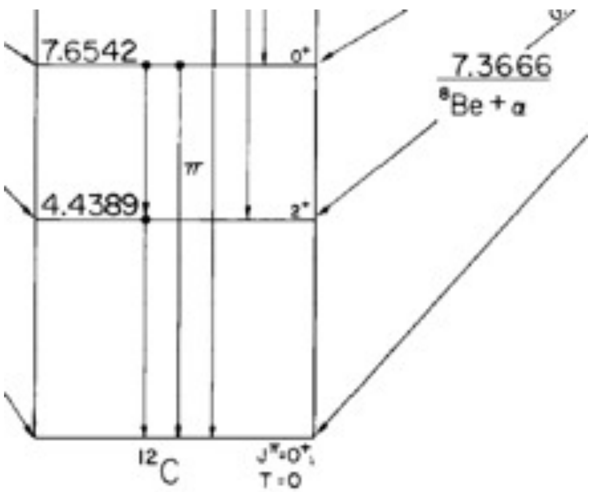

Resonances
Excited states
nucleon(s)
decay threshold
Electromagnetic
decays only
GS
TANTALUM
10 000
1000
100
10
1
TOTAL CROSS SECTION - BARNS
4.29
10.36
13.9
20.7
24.2
36.7
39.4
0.1
1
10
100
NEUTRON ENERGY - ev

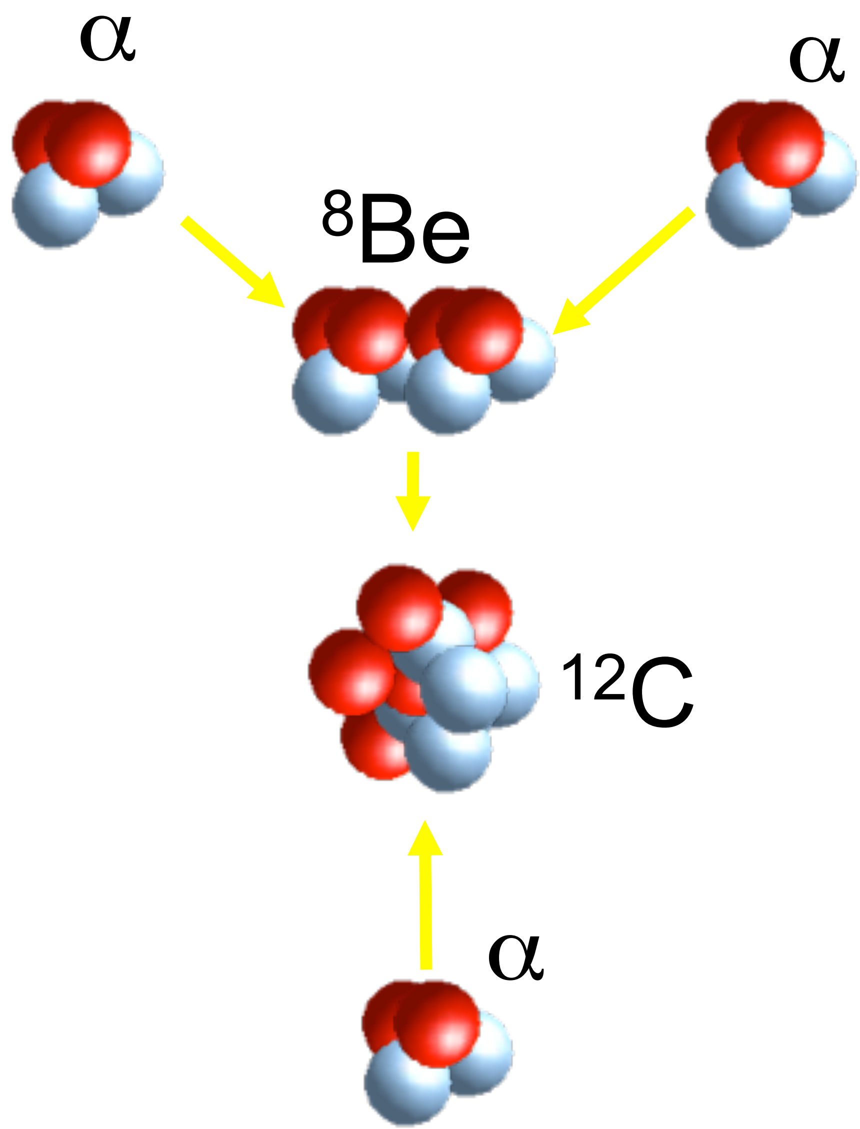
Practically unlimited yield of thermonuclear explosion is possible due to resonance in ^{5}He!



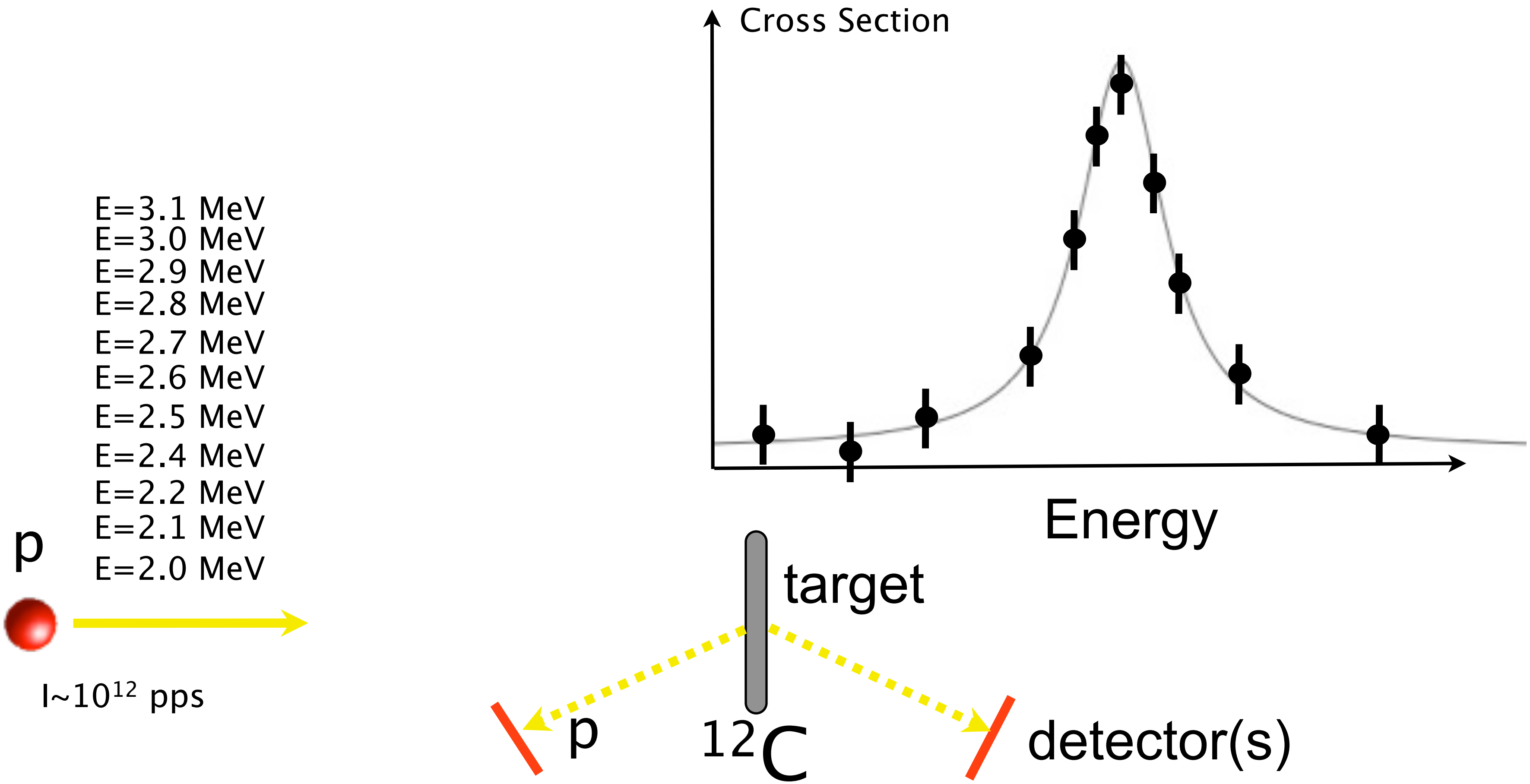
$$^3H + ^2H \rightarrow ^4He + n + 17.8\ MeV$$

Hoyle state in ^{12}C at 7.65 MeV is responsible for production of ^{12}C in red giants and ultimately for our existence



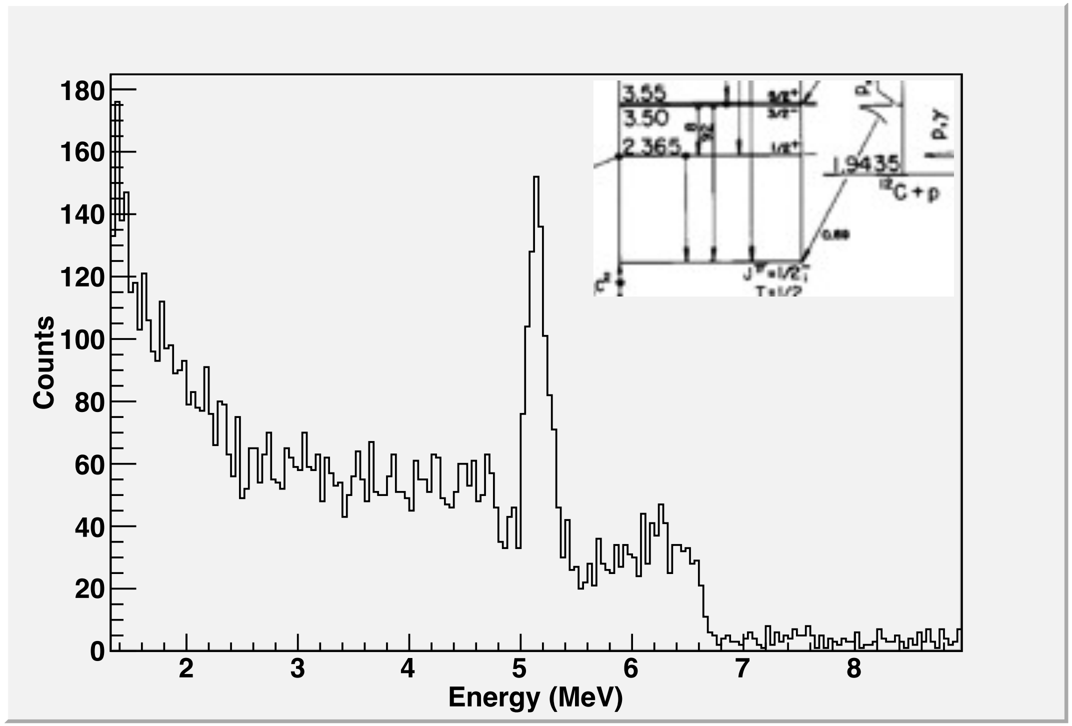


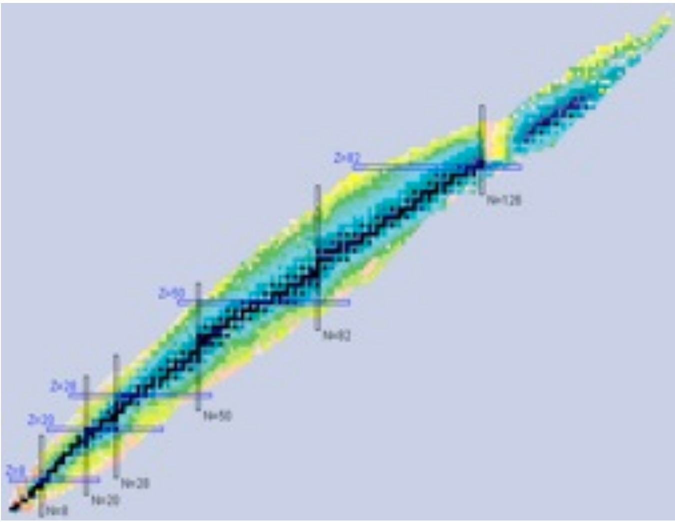
Observation of a resonance in an elastic scattering

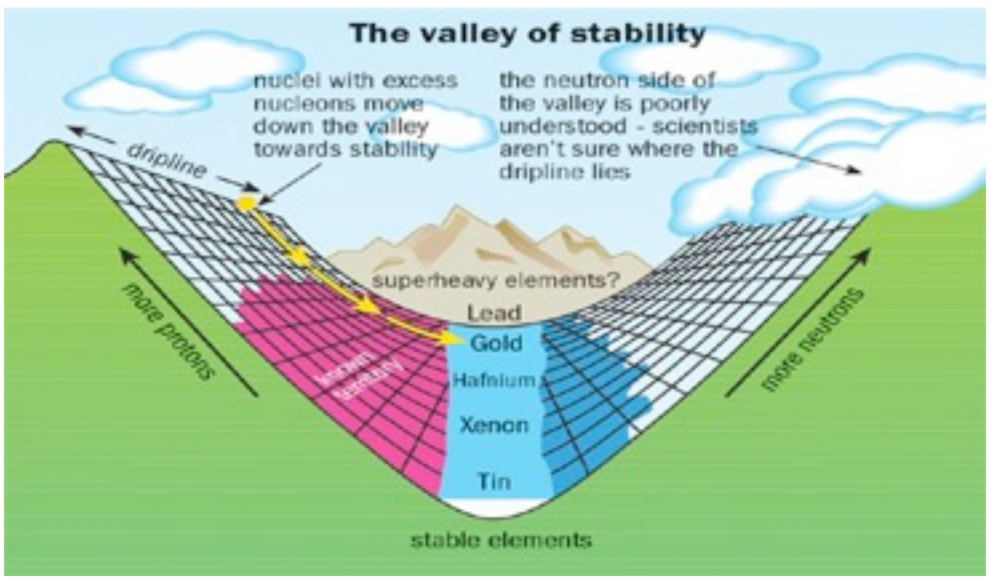


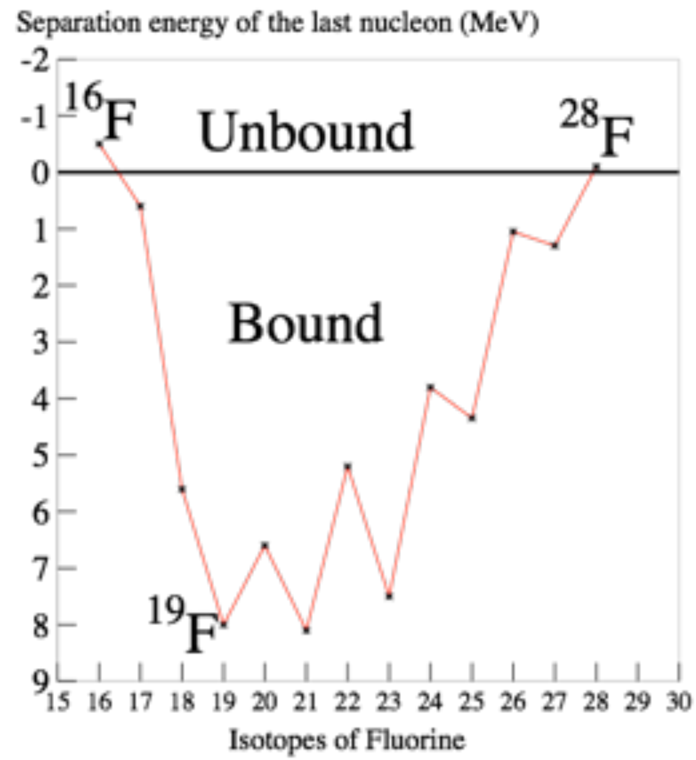
Excitation function for ^{12}C+p elastic scattering

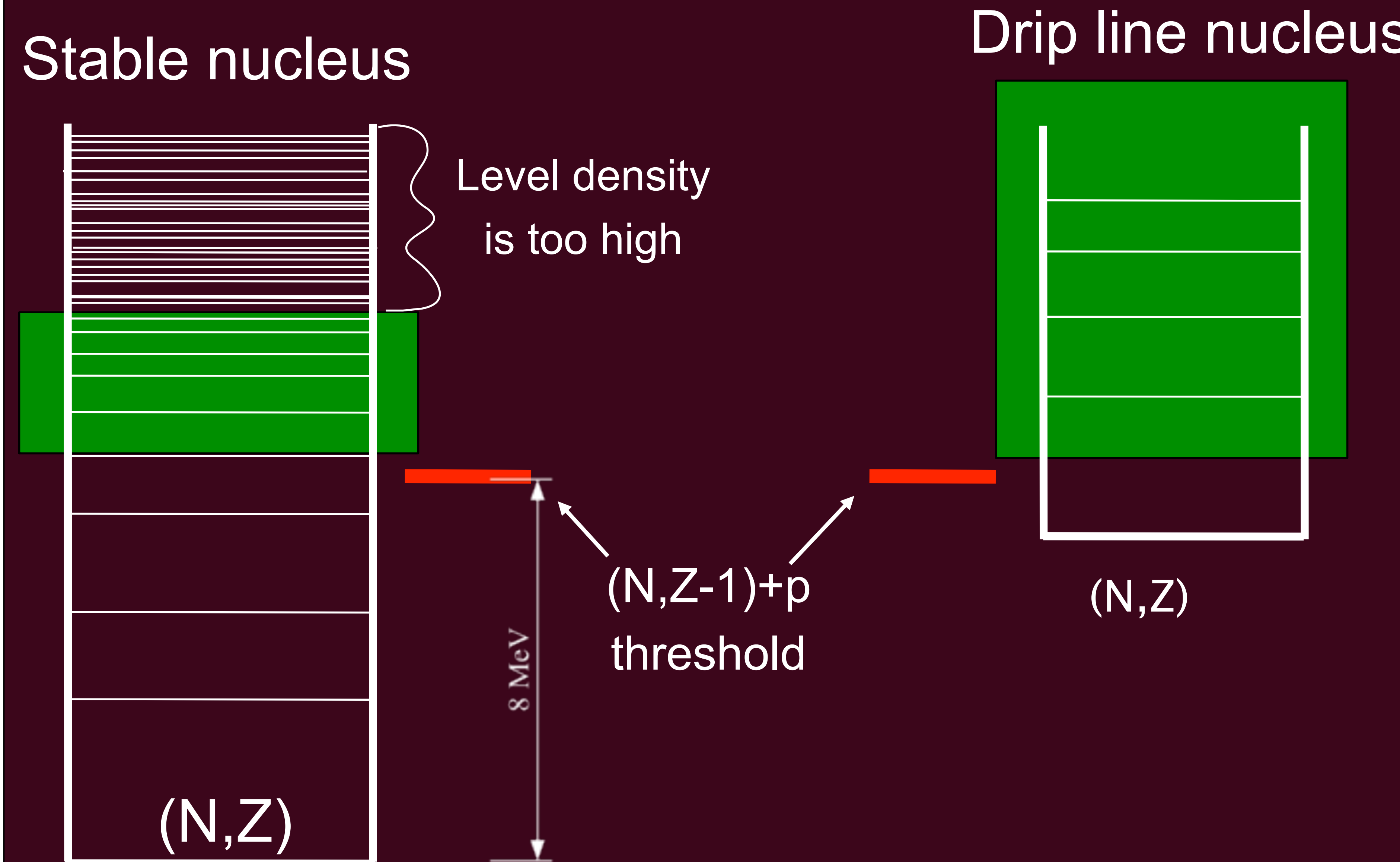
The spectrum is from the ReA3/ANASEN test run at NSCL.



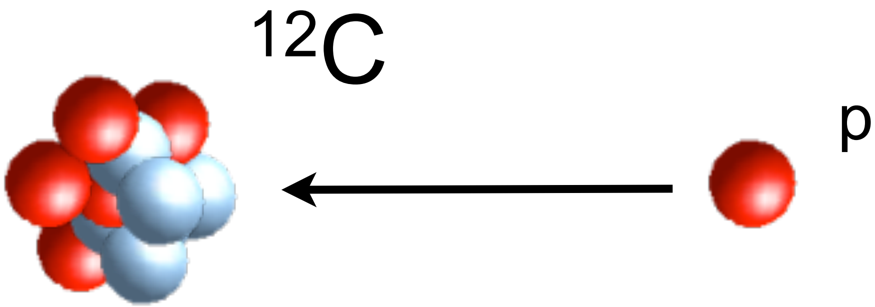



The valley of stability
nuclei with excess nucleons move down the valley towards stability
the neutron side of the valley is poorly understood - scientists aren't sure where the dripline lies
dripline
superheavy elements?
Lead
Gold
Hafnium
Xenon
Tin
more protons
more neutrons
stable elements


Separation energy of the last nucleon (MeV)
^{16}F
Unbound
^{28}F
Bound
^{19}F
-2
-1
0
1
2
3
4
5
6
7
8
9
15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
Isotopes of Fluorine


Stable nucleus
Drip line nucleus
Level density
is too high
(N,Z)
8 MeV
(N,Z-1)+p
threshold
(N,Z)

Learning about nuclear structure from resonances. R-matrix theory.



$$\nabla^2\Psi + \frac{2\mu}{\hbar^2}(E - V)\Psi = 0 \qquad \mu = \frac{mM}{m+M}$$

$$\Psi = \sum_{\ell} \frac{u(\ell)}{r} P_\ell(\cos\theta)$$

$$\frac{d^2 u_\ell}{dr^2} + \left\{ \frac{2\mu}{\hbar^2}(E - V(r)) - \frac{\ell(\ell+1)}{r^2} \right\} u_\ell = 0$$

R-matrix theory

$$u(r \to \infty) \approx A_\ell \sin(kr + \delta_\ell)$$

k - wave number: $p = k\hbar = \sqrt{2\mu E}$

phase shift: $\delta_\ell = f(E, V)$

Using Euler's formula $\sin(x) = \frac{e^{ix} - e^{-ix}}{2i}$

$$u(r \to \infty) \sim e^{-ikr} - e^{2i\delta} e^{ikr} = I - UO$$

$U = e^{2i\delta}$ collision (scattering) function

Collision matrix is related to the observable: the scattering cross section.

R-matrix theory

$$u(r \to \infty) \sim e^{-ikr} - e^{2i\delta} e^{ikr} = I - UO$$

$$U = e^{2i\delta}$$

$$\Psi(r, \theta, \varphi) = A\,[e^{ikz} + (1/r)\, f(\theta,\varphi) e^{ikr}]$$

$$\sigma(\theta, E) = |f(\theta, E)|^2$$

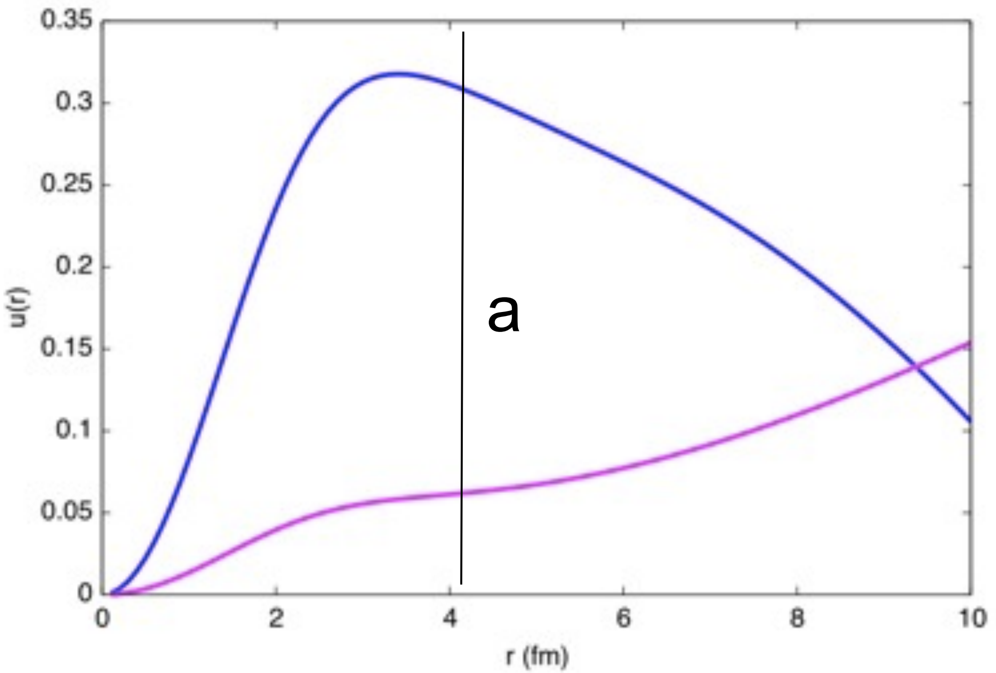
Straightforward manipulations can be used to show that:

$$f(\theta, E) = \frac{i}{2k} \sum_{\ell=0}^{\infty} (2\ell + 1)(1 - U_\ell) P_\ell(\cos(\theta))$$

Problem: How to relate the measured cross section to the properties of the wave function in the interior region.

R-matrix theory

On-resonance and off-resonance behavior of the interior wave function



$$R = \left(\frac{u_\ell}{\rho u'_\ell}\right)_{r=a}$$

$$\rho = kr$$

$$u = I - UO$$

$$R = \frac{I - UO}{\rho(I' - UO')}$$

R-matrix theory

Applying Green's theorem to Schroedinger eq. leads to

$$R = \left(\frac{u_\ell}{\rho u'_\ell}\right)_{r=a} = \sum_\lambda \frac{\gamma_\lambda^2}{E_\lambda - E} \qquad \gamma_\lambda = \sqrt{\frac{\hbar^2}{2\mu a}}u_\lambda(a)$$

reduced width amplitude

E_λ - Eigenvalues and

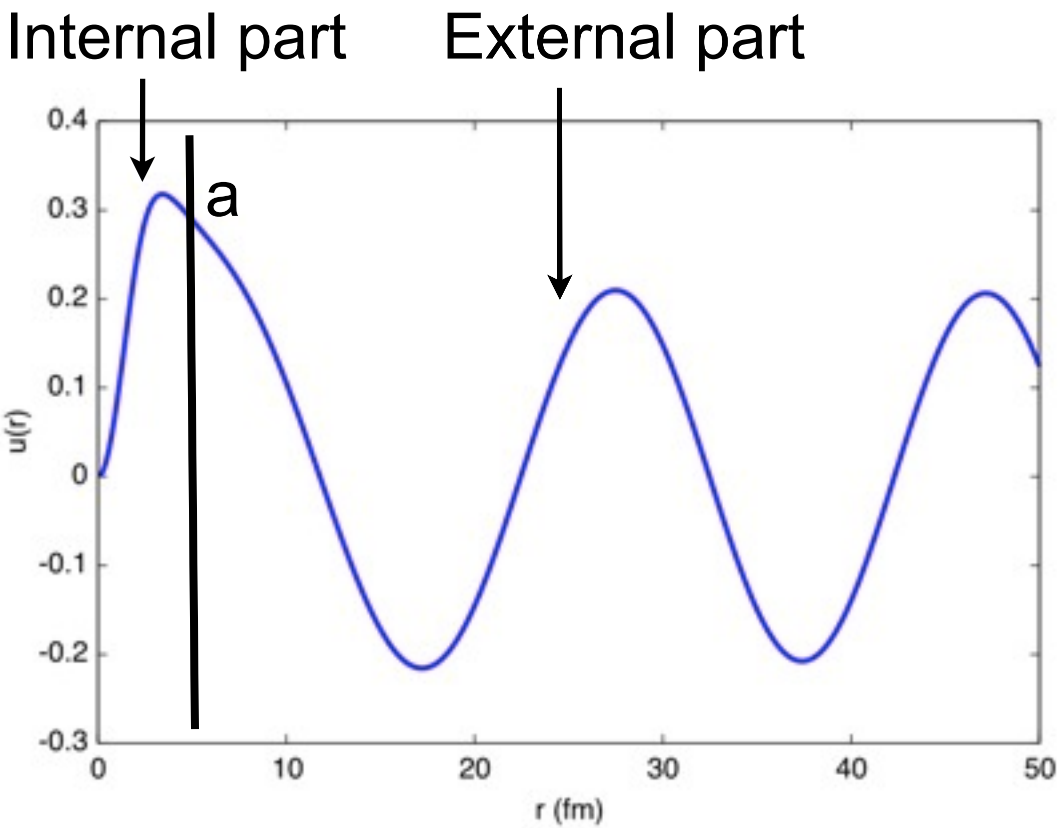
$u_\lambda(a)$ - eigenfunctions of Schroedinger eq. which satisfies

$$\frac{a}{u_\lambda(a)}\left(\frac{du_\lambda}{dr}\right)_{r=a} = B$$

boundary condition.

Interaction is unknown, so eigenvalues and values of eigenfunctions at **a** (channel radius) for EACH resonance are parameters of the theory. Other parameters are - channel radius "**a**" and boundary condition "**B**" (B is set independently for each partial wave).

R-matrix theory



In a nutshell:

1. The problem is split into two regions, internal and external.

2. Internal region, where interaction is important and unknown, is parametrized.

3. External part is described by asymptotic behavior of the wave functions under the assumption that there is no interaction (except for Coulomb!).

4. The phase shifts (collision functions) of the asymptotic wave functions are related to the R-function.

R-matrix theory

External wave function, assuming spinless but charged particles.

$I = (G - iF)e^{i\omega}$ Incoming wave function with Coulomb

$O = (G + iF)e^{-i\omega}$ Outgoing wave function with Coulomb

$\omega = \sum_{n=1}^{\ell} \tan^{-1} \frac{\eta}{n}$ Recall that $R = \frac{I - UO}{\rho(I' - UO')}$ and $U = e^{2i\delta}$

and the expression for the phase shift can be found:

$$\delta_\ell = \tan^{-1}\left(\frac{R_\ell P_\ell}{1 - R_\ell(S_\ell - B)}\right) - \phi_\ell + \omega_\ell$$

$P_\ell = \frac{ka}{F_\ell^2 + G_\ell^2}$ penetrability factor

$\phi_\ell = tan^{-1} \frac{F_\ell}{G_\ell}$ hard sphere phase shift

$S_\ell = \frac{FF' + GG'}{F^2 + G^2}$ ka shift factor

R-matrix theory

If cross section is dominated by an isolated resonance:

$$R \approx \frac{\gamma_\lambda^2}{E_\lambda - E} \qquad \delta_\ell = \tan^{-1}\left(\frac{\gamma_\ell^2 P_\ell}{E_\lambda - E - \gamma_\ell^2 (S_\ell - B)}\right) - \phi_\ell + \omega_\ell$$

Since $\sigma \sim |1 - e^{2i\delta}|^2$ CS is maximum when δ_ℓ = 90°

and it is 1/2 of the maximum when δ_ℓ = 45°

$E_r = E_\lambda - \gamma_\ell^2 (S_\ell(E_r) - B)$ Observed resonance energy

$\Gamma = 2P_\ell(E_r)\gamma^2$ Formal resonance width

$\Gamma = \frac{2P_\ell(E_r)\gamma^2}{1 + \gamma^2 \frac{dS(E_r)}{dE}}$ Observed resonance width

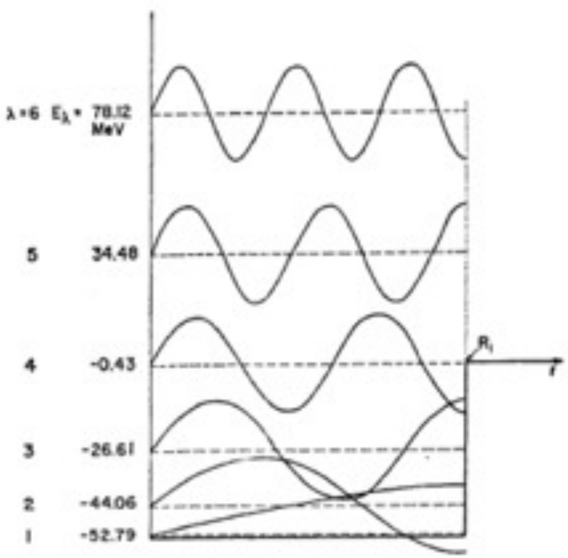
R-matrix theory

$$\gamma_\lambda = \sqrt{\frac{\hbar^2}{2\mu a}}\, u_\lambda(a)$$

reduced width amplitude is the parameter that is directly related to the structure of the specific resonance

So, what we should compare it to?

It is easy to show that the reduced width in the trivial case of a square well potential with radius **a** is:



$$u \sim \sin(Kr)$$

$$K = \frac{\sqrt{2\mu(E-V)}}{\hbar}$$

For zero boundary condition B=0

$$\gamma_{sw}^2 = \frac{\hbar^2}{\mu a^2}$$

R-matrix theory

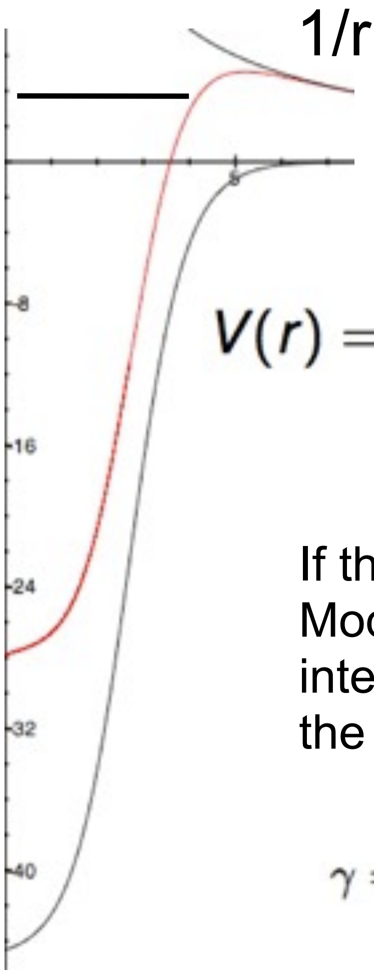
Square well reduced width is often used as a convenient measure of how "single particle" the level is.

$\theta_\lambda^2 = \frac{\gamma_\lambda^2}{\gamma_{sw}^2}$ dimensionless reduced width.

There is no need to use 3/2 factor introduced by Wigner before the Shell Model was discovered.

More quantitative measure of resonance's "purity" is provided by exact solution of Schroedinger equation with a more realistic potential. Woods-Saxon form is commonly used.

R-matrix theory



$$\theta_\lambda^2 = \frac{\gamma_\lambda^2}{\gamma_{WS}^2}$$

More accurate dimensionless reduced width is determined using Woods-Saxon potential reduced width amplitude

$$V(r) = \frac{-V_0}{1+e^{\frac{r-R}{a}}}$$

If the wave function of the compound state is calculated (using Shell Model, for ex.) then the reduced width can be related to the overlap integral between the channel wave function and the wave function of the compound state calculated at the surface of radius **a**.

$$\gamma = \left(\frac{\hbar^2}{2\mu a}\right)^{1/2} \int [\psi(p) \times \psi(^{12}C)]\Psi(^{13}N)dS_c$$

R-matrix theory

R-matrix vs Exact solution of Schroedinger equation

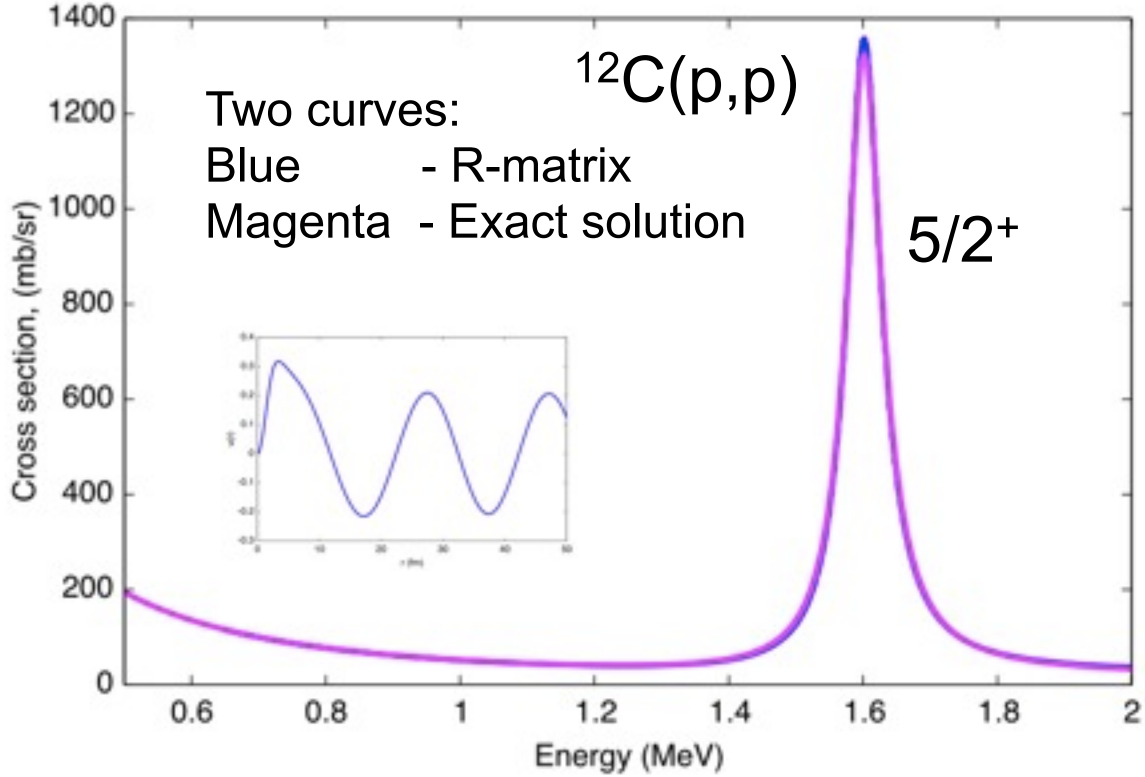
$B = -2.0$
$a = 4.2$ fm

$E_\lambda = 1.635$ MeV
$\gamma_\lambda = 1.4\ \mathrm{MeV}^{1/2}$

$E_{obs} = 1.603$ MeV
$\Gamma_{obs} = 64$ keV

W-S potential parameters:

$V = -54.4$ MeV
$a = 0.662$ fm
$r_o = 1.26$ fm
$V_{so} = 6.4$ MeV



R-matrix theory

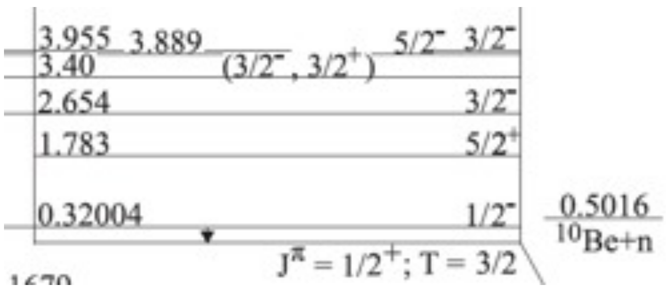
Dependence on the channel radius and boundary condition

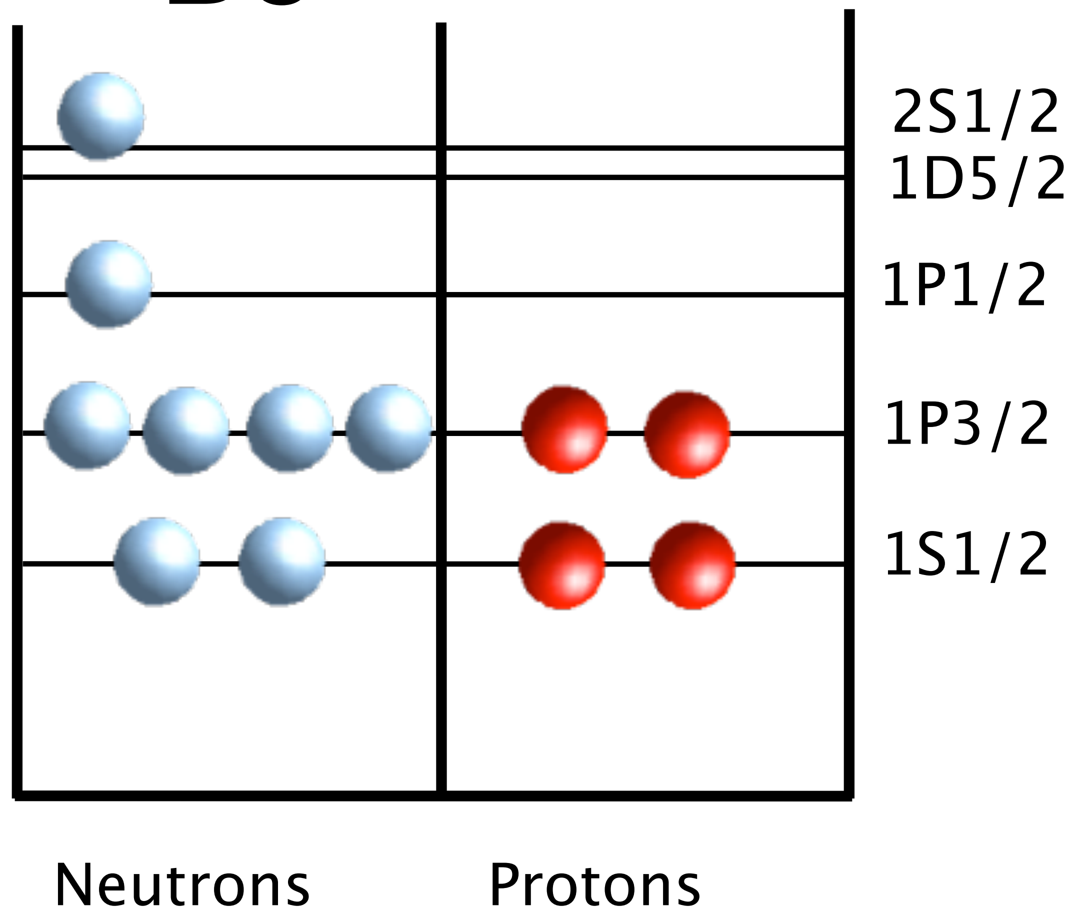
a fm	B	E_λ MeV	γ_λ $MeV^{1/2}$	E_{obs} MeV	Γ_{obs} keV	θ^2_{sw}
4.2	-2.0	1.635	1.4	1.603	64	0.76
4.2	0.0	-2.285	1.4	1.603	64	0.76
4.2	-1.0	-0.325	1.4	1.603	64	0.76
5.2	-2.0	1.685	0.75	1.603	64	0.33
6.2	-2.0	1.675	0.48	1.603	64	0.19
3.95	-2.008	1.603	1.7	1.603	64	1.0

$R_{^{12}C} + R_p = 2.61 + 0.84 = 3.45$ fm

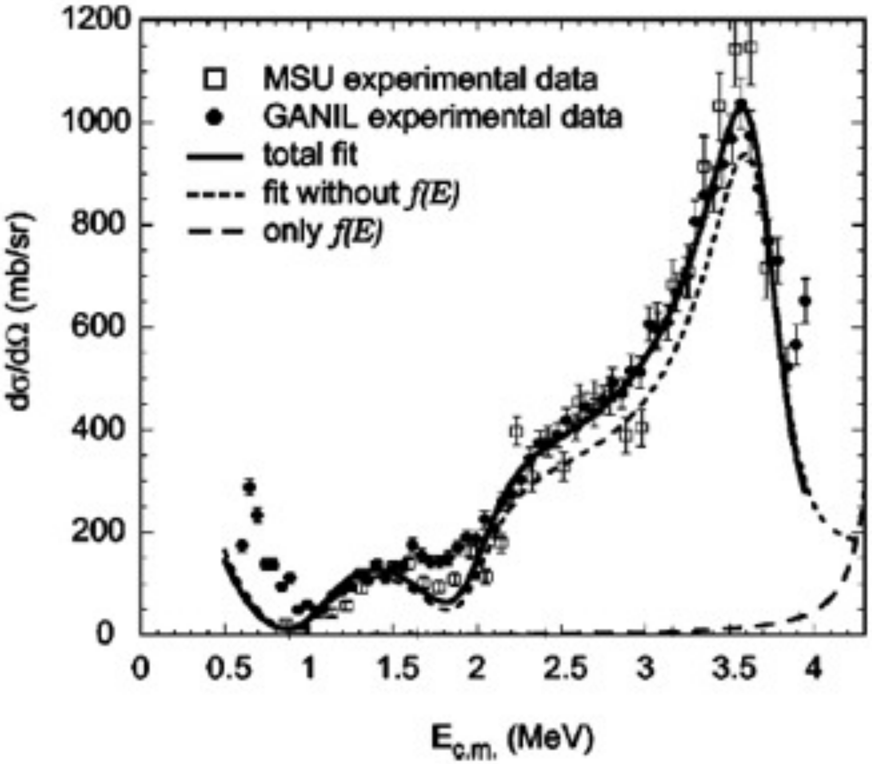
Prescription that usually works well: $\mathbf{a = 1.4 * A^{1/3} + 0.84}$

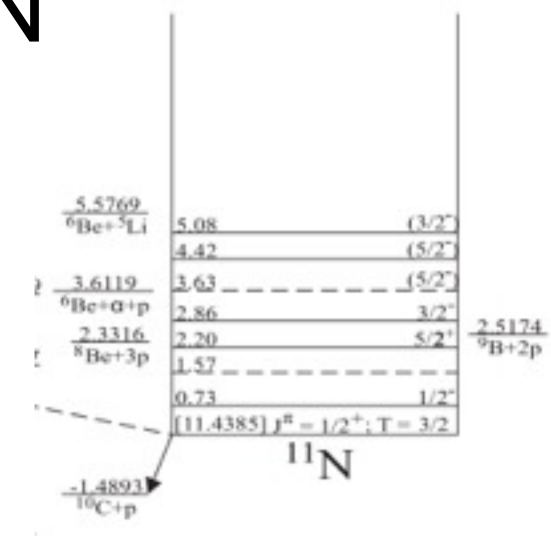
^{11}Be

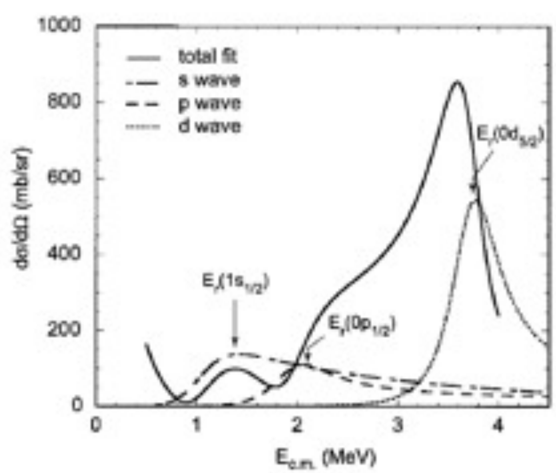

3.955 3.889 5/2⁻ 3/2⁻
3.40 (3/2⁻, 3/2⁺)
2.654 3/2⁻
1.783 5/2⁺
0.32004 1/2⁻
0.5016
^{10}Be+n
$J^{\pi} = 1/2^{+}; T = 3/2$


2S1/2
1D5/2
1P1/2
1P3/2
1S1/2
Neutrons
Protons

Structure of ^{11}N







L. Axelsson, et al., PRC 54, R1511 (1996)

K. Markenroth, et al., PRC 62, 034308 (2000)

R-matrix theory

Multi-level, multi channel problem for charged particles with non-zero spin.

$$A_{\alpha's'\nu',\alpha s\nu}(\Omega_{\alpha'})=\frac{\pi^{\frac{1}{2}}}{k_\alpha}\Big[-C_{\alpha'}(\theta_{\alpha'})\delta_{\alpha's'\nu',\alpha s\nu}$$
$$+i\sum_{JMll'm'}(2l+1)^{\frac{1}{2}}(sl\nu0|JM)(s'l'\nu'm'|JM)$$
$$\times T_{\alpha's'l',\alpha sl}{}^{J}Y_{m'}{}^{(l')}(\Omega_{\alpha'})\Big],\quad(2.3)$$

where

$$T_{\alpha's'l',\alpha sl}{}^{J}=e^{2i\omega_{\alpha l}}\delta_{\alpha's'l',\alpha sl}-U_{\alpha's'l',\alpha sl}{}^{J}.$$

In performing the absolute squaring operation, one introduces the two sets of summing integers

$$\{J_1M_1l_1l_1'm_1'\}\quad\text{and}\quad\{J_2M_2l_2l_2'm_2'\}$$

for the single set of (2.3), and thereby obtains for (2.1)

$$(2s+1)\frac{k_\alpha{}^2}{\pi}d\sigma_{\alpha s,\alpha's'}d\Omega_{\alpha'}=(2s+1)|C_{\alpha'}(\theta_{\alpha'})|^2\delta_{\alpha's',\alpha s}$$
$$+\sum_{\substack{J_1J_2M_1M_2\\ l_1l_2l_1'l_2'\\ \nu\nu'm_1'm_2'}}(2l_1+1)^{\frac{1}{2}}(2l_2+1)^{\frac{1}{2}}(sl_1\nu0|J_1M_1)$$
$$\times(sl_2\nu0|J_2M_2)(s'l_1'\nu'm_1'|J_1M_1)(s'l_2'\nu'm_2'|J_2M_2)$$
$$\times(T_{\alpha's'l_1',\alpha sl_1}{}^{J_1}Y_{m_1'}{}^{(l_1')}(\Omega_{\alpha'}))$$
$$\times(T_{\alpha's'l_2',\alpha sl_2}{}^{J_2}Y_{m_2'}{}^{(l_2')}(\Omega_{\alpha'}))^*$$
$$-\sum_{\substack{JMll'\\ m'\nu\nu'}}(2l+1)^{\frac{1}{2}}(sl\nu0|JM)(s'l'\nu'm'|JM)$$
$$\times\delta_{\alpha's'\nu',\alpha s\nu}2\,\mathrm{Re}\big[iT_{\alpha's'l',\alpha sl}{}^{J}Y_{m'}{}^{(l')}(\Omega_{\alpha'})C_{\alpha'}(\theta_{\alpha'})\big].$$
$$(2.4)$$

$$R\rightarrow R_{\alpha s\ell,\alpha's'\ell'}=\sum_{\lambda}\frac{\gamma_{\lambda c}\gamma_{\lambda c'}}{E_\lambda-E}$$

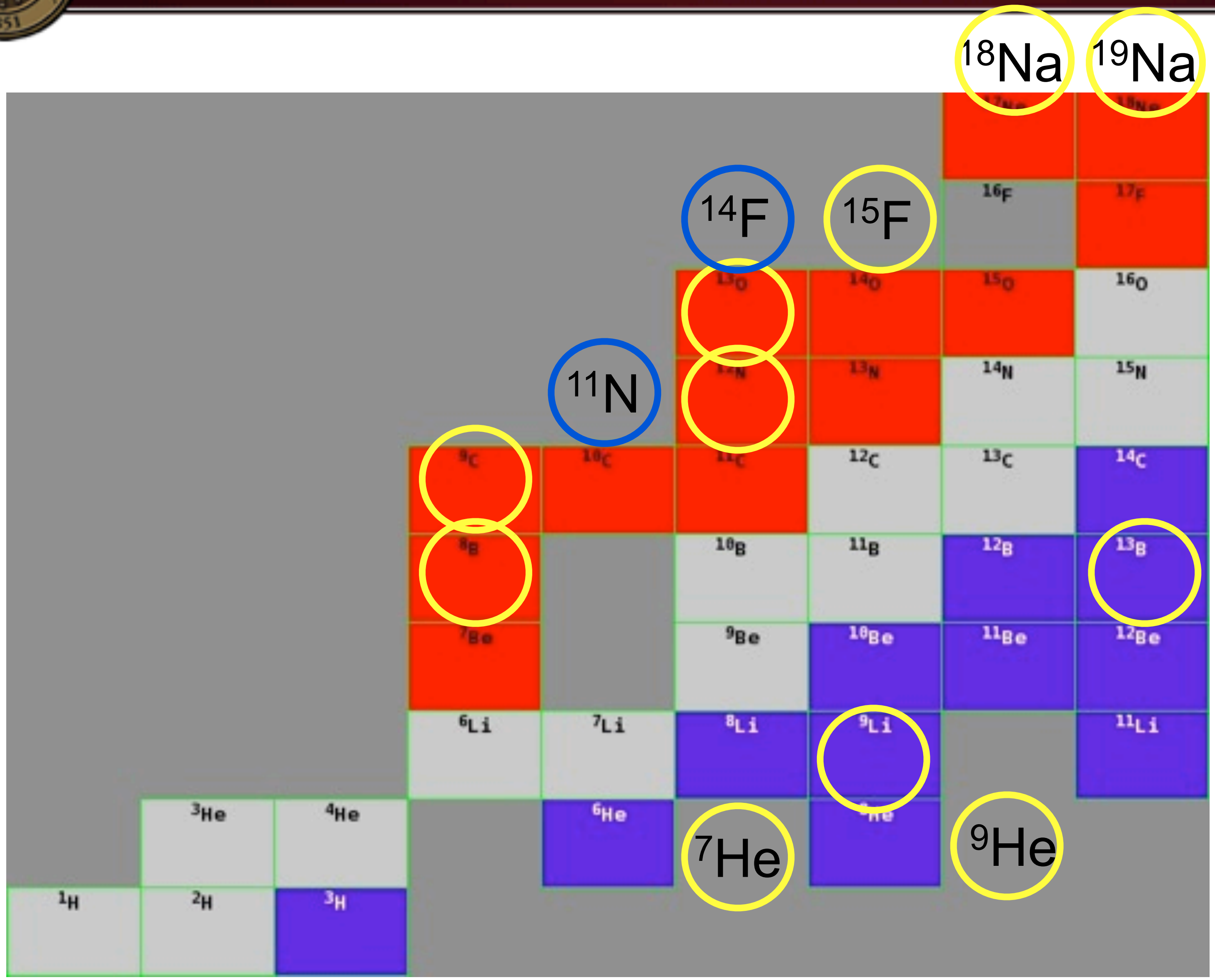
$$U\rightarrow U_{\alpha s\ell,\alpha's'\ell'}$$

$$\sigma_{\alpha\alpha}\sim C^2+N^2+C*N$$

$$\sigma_{\alpha\alpha'}\sim N^2$$

Available codes: SAMMY (Oak Ridge)
AZURE (Notre Dame)
MinRmatrix (FSU)

A.M. Lane and R.G. Thomas, Rev. of Mod. Phys., 30 (1958) 257


18Na
19Na
14F
15F
16F
17F
16O
14O
15O
11N
14N
15N
13N
9C
10C
12C
13C
14C
8B
10B
11B
12B
13B
7Be
9Be
10Be
11Be
12Be
6Li
7Li
8Li
9Li
11Li
3He
4He
6He
7He
9He
1H
2H
3H