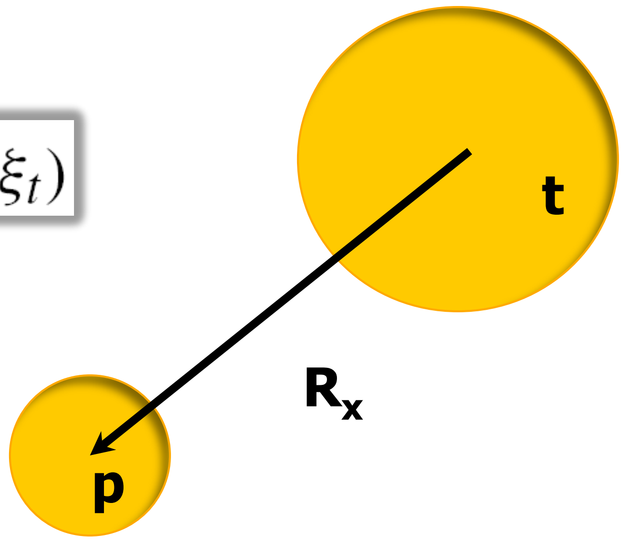
Scattering theory: multi-channel

Multichannel coupled equations

$$\boxed{H = H_{xp}(\xi_p) + H_{xt}(\xi_t) + \hat{T}_x(R_x) + \mathcal{V}_x(R_x, \xi_p, \xi_t)}$$



$$H_{xp}(\xi_p)\phi_{I_p}^{xp}(\xi_p) = \epsilon_{xp}\phi_{I_p}^{xp}(\xi_p),$$

$$H_{xt}(\xi_t)\phi_{I_t}^{xt}(\xi_t) = \epsilon_{xt}\phi_{I_t}^{xt}(\xi_t),$$

$$\mathcal{V}_x(R_x, \xi_p, \xi_t) = \sum_{i\in p, j\in t} V_{ij}(\mathbf{r}_i - \mathbf{r}_j)$$

within the same partition, the Schrodinger equations becomes a coupled equation:

$$\boxed{[\hat{T}_{xL}(R_x) - E_{xpt}]\psi_\alpha(R_x) + \sum_{\alpha'} \hat{V}_{\alpha\alpha'}^{\text{prior}}\psi_{\alpha'}(R_{x'}) = 0.}$$

Integrated channel cross section

- channel cross section

$$\sigma_{xpt}(\theta) = \frac{1}{(2I_{p_i}+1)(2I_{t_i}+1)} \sum_{\mu_p\mu_t,\mu_{p_i}\mu_{t_i}} \left| \tilde{f}^{\,xpt}_{\mu_p\mu_t,\mu_{p_i}\mu_{t_i}}(\theta) \right|^2$$

- total outgoing non-elastic cross section

$$\begin{aligned}
\sigma_{xpt} &= 2\pi \int_0^{\pi} \mathrm{d}\theta \sin\theta \, \sigma_{xpt}(\theta) \\
&= \frac{\pi}{k_i^2} \frac{1}{(2I_{p_i}+1)(2I_{t_i}+1)} \sum_{J_{\mathrm{tot}}\pi L J \alpha_i} (2J_{\mathrm{tot}}+1) |\tilde{\mathbf{S}}^{J_{\mathrm{tot}}\pi}_{\alpha\alpha_i}|^2 \\
&= \frac{\pi}{k_i^2} \sum_{J_{\mathrm{tot}}\pi L J \alpha_i} g_{J_{\mathrm{tot}}} |\tilde{\mathbf{S}}^{J_{\mathrm{tot}}\pi}_{\alpha\alpha_i}|^2,
\end{aligned}$$

Reaction cross section

NSCL

- flux leaving the elastic channel (depends only on elastic S-matrix elements)

$$\sigma_R = \frac{\pi}{k_i^2}\frac{1}{(2I_{p_i}+1)(2I_{t_i}+1)}\sum_{J_{\rm tot}\pi\alpha_i}(2J_{\rm tot}+1)(1-|\mathbf{S}^{J_{\rm tot}\pi}_{\alpha_i\alpha_i}|^2)$$
$$= \frac{\pi}{k_i^2}\sum_{J_{\rm tot}\pi\alpha_i} g_{J_{\rm tot}}(1-|\mathbf{S}^{J_{\rm tot}\pi}_{\alpha_i\alpha_i}|^2), \text{ similarly.}$$

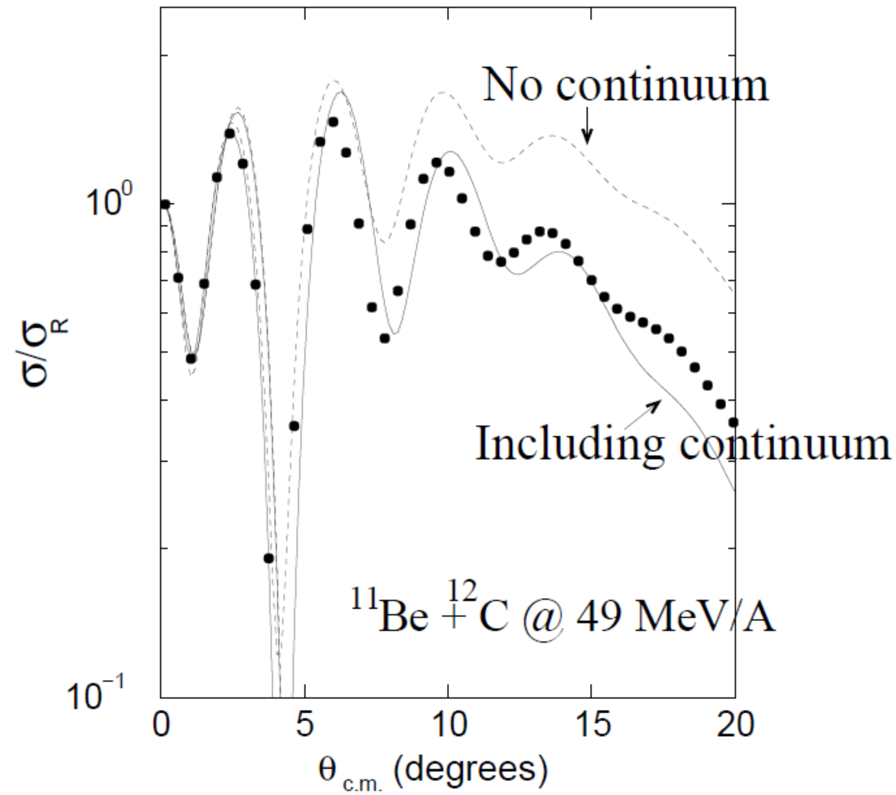
- the total cross section is elastic plus reaction cross sections

$$\sigma_{\rm tot} = \sigma_R + \sigma_{\rm el}$$
$$= \frac{2\pi}{k_i^2}\frac{1}{(2I_{p_i}+1)(2I_{t_i}+1)}\sum_{J_{\rm tot}\pi\alpha_i}(2J_{\rm tot}+1)[1-\mathrm{Re}\,\mathbf{S}^{J_{\rm tot}\pi}_{\alpha_i\alpha_i}]$$

- absorption cross section

$$\sigma_A = \sigma_R - \sum_{xpt\neq x_ip_it_i}\sigma_{xpt}$$

elastic scattering: multi-channel



Courtesy of Antonio Moro

detailed balance

Consequence of hermiticity: S-matrix is unitary

$$\sum_{\alpha} \tilde{\mathbf{S}}^{*}_{\alpha\alpha_i} \tilde{\mathbf{S}}_{\alpha\alpha'_i} = \delta_{\alpha_i\alpha'_i},$$

Even if the S-matrix is not unitary, it may be that:

$$|\tilde{\mathbf{S}}_{\alpha\alpha_i}|^2 = |\tilde{\mathbf{S}}_{\alpha_i\alpha}|^2,$$

Symmetry condition is sufficient for detailed balance:

$$\boxed{\sigma_{x_ip_it_i:xpt} = \frac{k_i^2(2I_{p_i}+1)(2I_{t_i}+1)}{k^2(2I_p+1)(2I_t+1)}\sigma_{xpt:x_ip_it_i}.}$$

$$\sigma_{xpt:x_ip_it_i} = \frac{\pi}{k_i^2}\frac{1}{(2I_{p_i}+1)(2I_{t_i}+1)}\sum_{J_{\mathrm{tot}}\pi\alpha\alpha_i}(2J_{\mathrm{tot}}+1)|\tilde{\mathbf{S}}^{J_{\mathrm{tot}}\pi}_{\alpha\alpha_i}|^2.$$

Scattering: Integral formulation

Integral forms and T-matrix approach

NSCL

(E - T) ψ=Vψ

G=(E-T)$^{-1}$

Lippmann-Schwinger equation

$$\psi = \phi + \hat{G}^{+}\Omega$$
$$= \phi + \hat{G}^{+} V \psi ,$$

ϕ is incoming free wave
(only non zero for elastic channel)

ψ is full wavefunction

Partial wave T-matrix $\mathbf{T} = -\frac{2\mu}{\hbar^2 k}\langle \phi^{(-)} | V | \psi \rangle \equiv -\frac{2\mu}{\hbar^2 k} \int \phi(R) V(R) \psi(R) \mathrm{d}R.$

Vector T-matrix

$$\mathbf{T}(\mathbf{k}', \mathbf{k}) = \langle \mathrm{e}^{\mathrm{i}\mathbf{k}'\cdot\mathbf{R}} | V | \Psi(\mathbf{R}; \mathbf{k}) \rangle .$$

Scattering amplitude

$$f(\mathbf{k}'; \mathbf{k}) = -\frac{\mu}{2\pi\hbar^2} \mathbf{T}(\mathbf{k}', \mathbf{k})$$

two potential formula: definitions

Consider your potential can be split into two parts: $U=U_1+U_2$

Free:	$[E-T]\phi=0$	$\hat{G}_0^+=[E-T]^{-1}$	$\phi=F$
Distorted:	$[E-T-U_1]\chi=0$	$\chi=\phi+\hat{G}_0^+U_1\chi$	$\chi\rightarrow\phi+\mathbf{T}^{(1)}H^+$
Full:	$[E-T-U_1-U_2]\psi=0$	$\psi=\phi+\hat{G}_0^+(U_1+U_2)\psi$	$\psi\rightarrow\phi+\mathbf{T}^{(1+2)}H^+$

two potential formula: derivation 1

Free:	$[E-T]\phi = 0$	$\hat{G}_0^+ = [E-T]^{-1}$	$\phi = F$
Distorted:	$[E-T-U_1]\chi = 0$	$\chi = \phi + \hat{G}_0^+ U_1 \chi$	$\chi \to \phi + \mathbf{T}^{(1)} H^+$
Full:	$[E-T-U_1-U_2]\psi = 0$	$\psi = \phi + \hat{G}_0^+ (U_1+U_2)\psi$	$\psi \to \phi + \mathbf{T}^{(1+2)} H^+$

$$
\begin{aligned}
-\frac{\hbar^2 k}{2\mu}\mathbf{T}^{(1+2)} &= \int \phi(U_1 + U_2)\psi \, \mathrm{d}R \\
&= \int (\chi - \hat{G}_0^+ U_1 \chi)(U_1 + U_2)\psi \, \mathrm{d}R \\
&= \int \Big[\chi(U_1+U_2)\psi - (\hat{G}_0^+ U_1 \chi)(U_1+U_2)\psi\Big] \mathrm{d}R \,.
\end{aligned}
$$

two potential formula: derivation 2

Free:	$[E-T]\phi = 0$	$\hat{G}_0^+ = [E-T]^{-1}$	$\phi = F$
Distorted:	$[E-T-U_1]\chi = 0$	$\chi = \phi + \hat{G}_0^+ U_1 \chi$	$\chi \to \phi + \mathbf{T}^{(1)} H^+$
Full:	$[E-T-U_1-U_2]\psi = 0$	$\psi = \phi + \hat{G}_0^+ (U_1+U_2)\psi$	$\psi \to \phi + \mathbf{T}^{(1+2)} H^+$

$$
\begin{aligned}
-\frac{\hbar^2 k}{2\mu}\mathbf{T}^{(1+2)} &= \int [\chi(U_1+U_2)\psi - \chi U_1 \hat{G}_0^+ (U_1+U_2)\psi]\, \mathrm{d}R \\
&= \int [\chi(U_1+U_2)\psi - \chi U_1(\psi-\phi)]\, \mathrm{d}R \\
&= \int [\phi U_1 \chi + \chi U_2 \psi]\, \mathrm{d}R \\
&= \langle \phi^{(-)} | U_1 | \chi \rangle + \langle \chi^{(-)} | U_2 | \psi \rangle .
\end{aligned}
$$

two potential formula: result

Free:	$[E-T]\phi = 0$	$\hat{G}_0^+ = [E-T]^{-1}$	$\phi = F$
Distorted:	$[E-T-U_1]\chi = 0$	$\chi = \phi + \hat{G}_0^+ U_1 \chi$	$\chi \to \phi + \mathbf{T}^{(1)} H^+$
Full:	$[E-T-U_1-U_2]\psi = 0$	$\psi = \phi + \hat{G}_0^+ (U_1+U_2)\psi$	$\psi \to \phi + \mathbf{T}^{(1+2)} H^+$

$$\mathbf{T}^{(1+2)} = \mathbf{T}^{(1)} + \mathbf{T}^{2(1)}$$

$$\boxed{\mathbf{T}^{2(1)} = -\frac{2\mu}{\hbar^2 k} \int \chi U_2 \psi \, \mathrm{d}R}$$

Remember the Coulomb and nuclear?

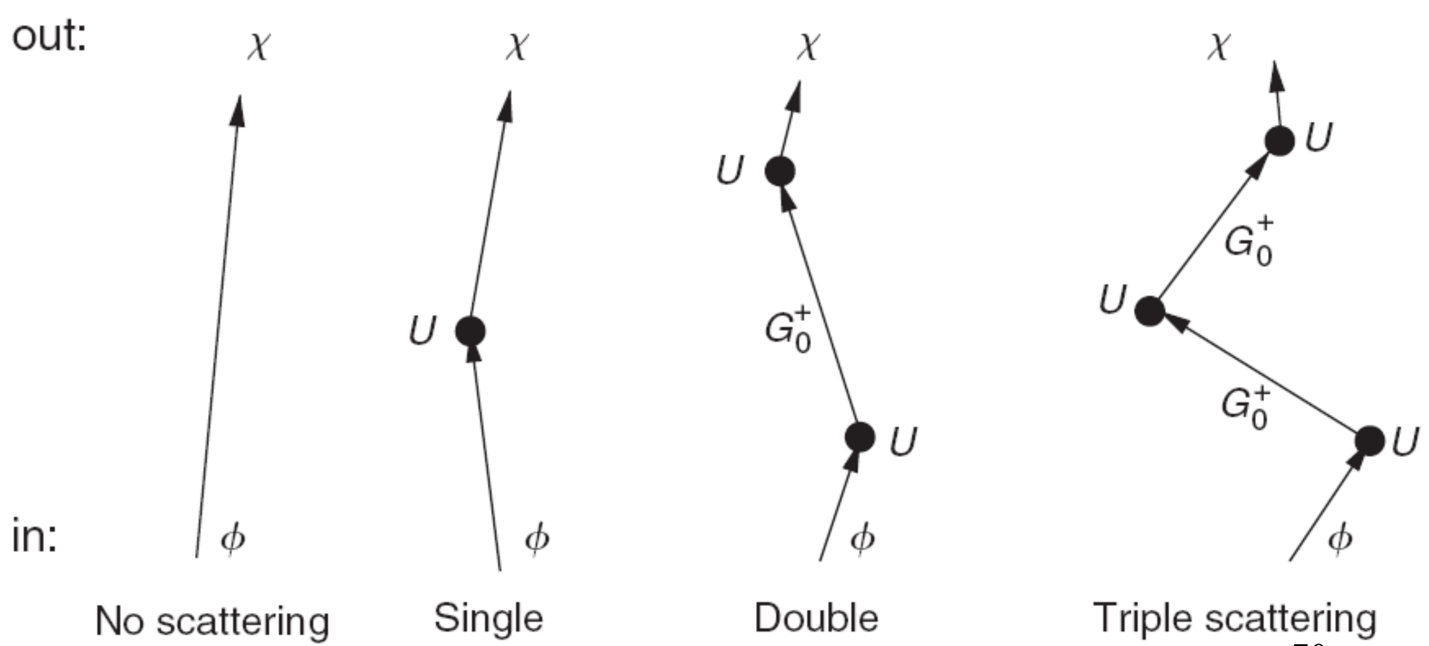
$$f_{nc}(\theta) = f_c(\theta) + f_n(\theta)$$

$$f_n(\theta) = \frac{1}{2ik} \sum_{L=0}^{\infty} (2L+1) P_L(\cos\theta) \mathrm{e}^{2i\sigma_L(\eta)} (\mathbf{S}_L^n - 1)$$

Born series

$$\chi = \phi + \hat{G}_0^+ U [\phi + \hat{G}_0^+ U [\phi + \hat{G}_0^+ U [\cdots]]]$$
$$= \phi + \hat{G}_0^+ U \phi + \hat{G}_0^+ U \hat{G}_0^+ U \phi + \hat{G}_0^+ U \hat{G}_0^+ U \phi \hat{G}_0^+ U \phi + \cdots ,$$

$$\mathbf{T} = -\frac{2\mu}{\hbar^2 k} \left[\langle \phi^{(-)} | U | \phi \rangle + \langle \phi^{(-)} | U \hat{G}_0^+ U | \phi \rangle + \cdots \right].$$



distorted wave Born approximation (DWBA)

$(E - T - U_1)\,\psi = U_2\psi$

$\psi = \chi + G_1 U_2 \psi$

Born series is truncated after the first term

$$\boxed{\mathbf{T}^{\mathrm{DWBA}} = \mathbf{T}^{(1)} - \frac{2\mu}{\hbar^2 k}\langle \chi^{(-)} | U_2 | \chi \rangle}$$

1st order DWBA: U_2 appears to first order

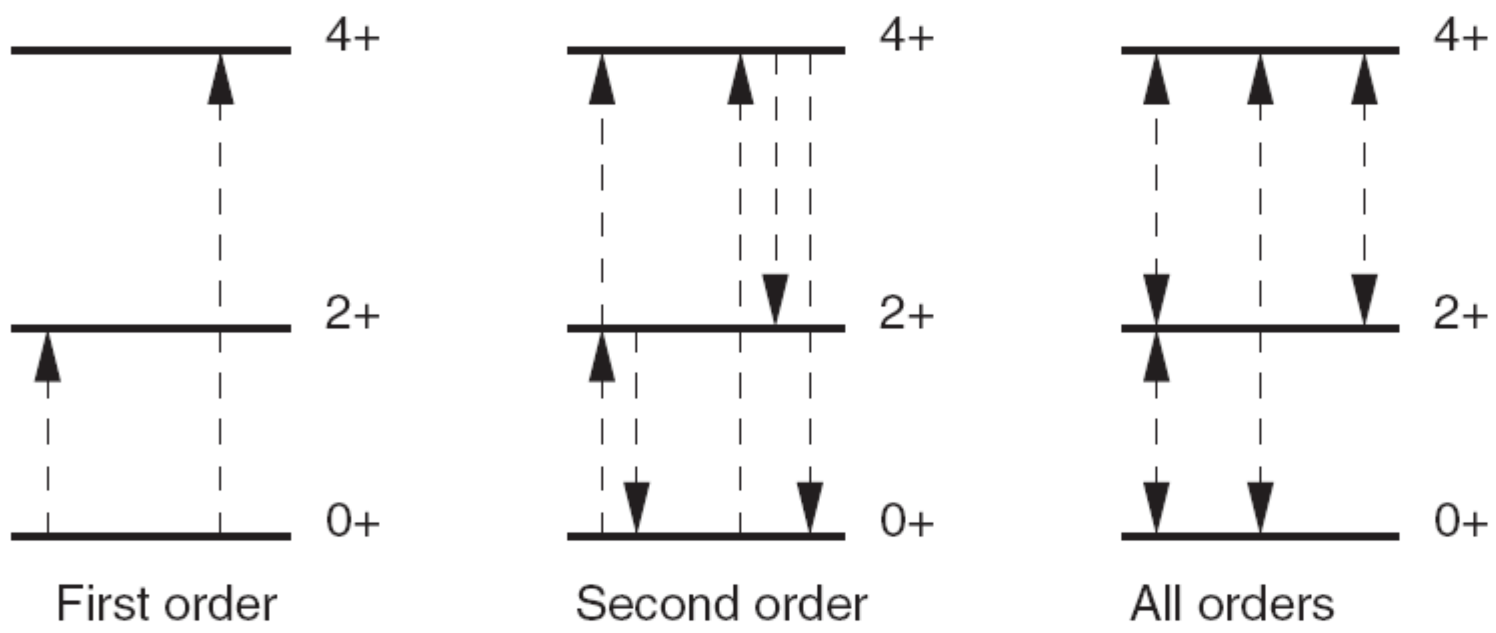
There is similarly a second-order DWBA expression

$$\mathbf{T}^{\mathrm{2nd-DWBA}}_{\alpha\alpha_i} = -\frac{2\mu_\alpha}{\hbar^2 k_\alpha}\left[\langle \chi^{(-)}_\alpha | U_2 | \chi_{\alpha_i} \rangle + \langle \chi^{(-)}_\alpha | U_2 \hat{G}^{+}_1 U_2 | \chi_{\alpha_i} \rangle\right].$$

U_2 appears to second order

multiple orders in DWBA

$$\mathbf{T}^{2\text{nd}-\text{DWBA}}_{\alpha\alpha_i} = -\frac{2\mu_\alpha}{\hbar^2 k_\alpha}\left[\langle\chi^{(-)}_\alpha|U_2|\chi_{\alpha_i}\rangle + \langle\chi^{(-)}_\alpha|U_2\hat{G}^+_1 U_2|\chi_{\alpha_i}\rangle\right].$$



QUIZ: Which of the following sentences is correct?

a) DWBA treats the transition potential U_2 perturbatively
b) DWBA treats the distorted waves perturbatively
c) DWBA treats the full projectile-target interaction perturbatively
d) DWBA is not a perturbative theory

T-matrix Lippmann-Schwinger Equation

Define a transition matrix (t-matrix) such that: $\langle \phi_{k'} | t | \phi_k \rangle = \langle \phi_{k'} | V | \psi_k^+ \rangle$

Remember the Born series?
$$\begin{aligned} \psi_k^+ &= \phi_k + G^+ V (\phi_k + G^+ V \psi_k^+) \\ &= \phi_k + G^+ V \phi_k + G^+ V G^+ V (\phi_k + G^+ V \psi_k^+) \\ &= \left(1 + \sum_{n=1}^{\infty} (G^+ V)^n \right) \phi_k \end{aligned}$$

Multiply by: $\langle \phi_{k'} | V$

and we can obtain an operator form of the equation in terms of the t-matrix
$$t = V\left(1 + \sum_{n=1}^{\infty} (G^+ V)^n\right)$$

$$t = V + V G^+ t$$

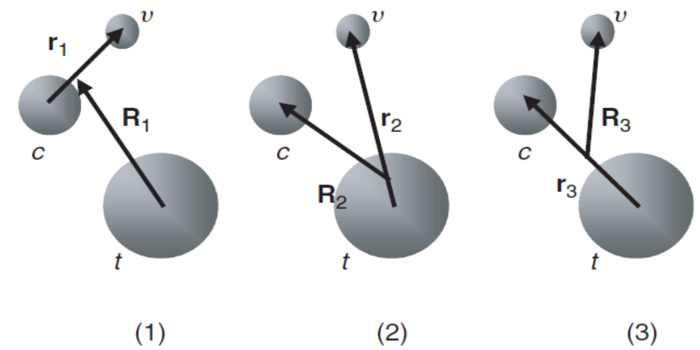
often used in few-body methods

Wong, Introduction to Nuclear Physics, Wiley

Three-body methods

Three-body methods in direct reactions

$$\Psi = \sum_{n=1}^{3} \Psi^{(n)}(\mathbf{r}_n, \mathbf{R}_n)$$



3 jacobi coordinate sets

3-body Hamiltonian for the problem:

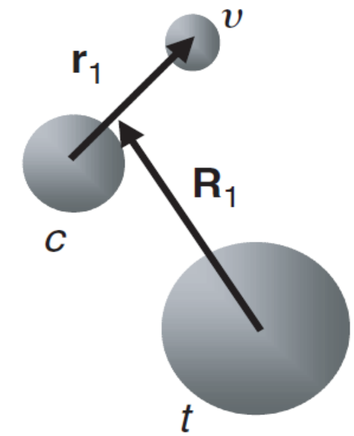
$$H_{3b} = \hat{T} + V_{vc} + V_{vt} + V_{ct}$$

Faddeev Equations:

$$\boxed{\begin{aligned}
(E - T_1 - V_{vc})\Psi^{(1)} &= V_{vc}(\Psi^{(2)} + \Psi^{(3)}) \\
(E - T_2 - V_{ct})\Psi^{(2)} &= V_{ct}(\Psi^{(3)} + \Psi^{(1)}) \\
(E - T_3 - V_{tv})\Psi^{(3)} &= V_{tv}(\Psi^{(1)} + \Psi^{(2)})
\end{aligned}}$$

reduction to one jacobi set

$$[H_{3b} - E]\Psi^{(1)}(\mathbf{r}_1, \mathbf{R}_1) = 0$$



Expand wfn in eigenstates of projectile's internal Hamiltonian:

$$\boxed{\Psi^{(1)}_{\mathbf{K}_0}(\mathbf{r}_1, \mathbf{R}_1) = \sum_{p=1}^{n_b} \phi_p(\mathbf{r}_1)\psi_p(\mathbf{R}_1) + \int \mathrm{d}\mathbf{k}\; \phi_{\mathbf{k}}(\mathbf{r}_1)\psi^{\mathbf{k}}_{\mathbf{K}}(\mathbf{R}_1)}$$

Expand in partial waves:

$$\phi^M_{(p,k)}(\mathbf{r}) = \frac{u_{(p,k)}(r)}{r}\left[\left[Y_\ell(\hat{\mathbf{r}}) \otimes \mathcal{X}_s\right]_j \otimes \mathcal{X}_{I_c}\right]_{I_p M}$$

$$H_{proj}\phi_p = \varepsilon\phi_p$$

Radial wavefunctions for projectile:

$$\left[-\frac{\hbar^2}{2\mu_{vc}}\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2} - \frac{\ell(\ell+1)}{r^2}\right) + V_{vc}(r) - \epsilon\right]u_{(p,k)}(r) = 0$$

bound states with $\epsilon_p < 0$

continuum states with energy $\epsilon_k > 0$

$$\boxed{E_{\mathrm{cm}} + \epsilon_0 = E = \frac{\hbar^2 k^2}{2\mu_{vc}} + \frac{\hbar^2 K^2}{2\mu_{(vc)t}}}$$

continuum bins

average method

$$\tilde{u}_p(r) = \sqrt{\frac{2}{\pi N_p}} \int_{k_{p-1}}^{k_p} g_p(k) u_k(r)\, \mathrm{d}k$$

- non overlaping continuum intervals - continuum bins are orthogonal
  - square integrable

analytic form if potential is zero and l=0:

$$\tilde{u}_p(r) \propto \sin(k_p r) \frac{\sin((k_p - k_{p-1})r)}{r}$$

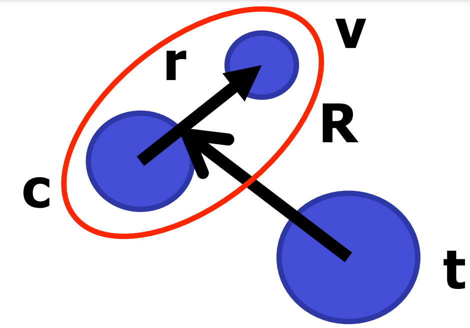
breakup: continuum discretized coupled channels

CDCC 3-body wavefunction:

$$\Psi^{\mathrm{CDCC}}(\mathbf{r},\mathbf{R}) = \sum_{p=0}^{N} \tilde{\phi}_p(\mathbf{r})\psi_p(\mathbf{R})$$

$$p = \{lsjI_cI_p; (k_{p-1}, k_p)\}$$

$$\boxed{(H_{3b} - E)\Psi^{\mathrm{CDCC}}(\mathbf{r},\mathbf{R}) = 0}$$



Coupled channel equations:

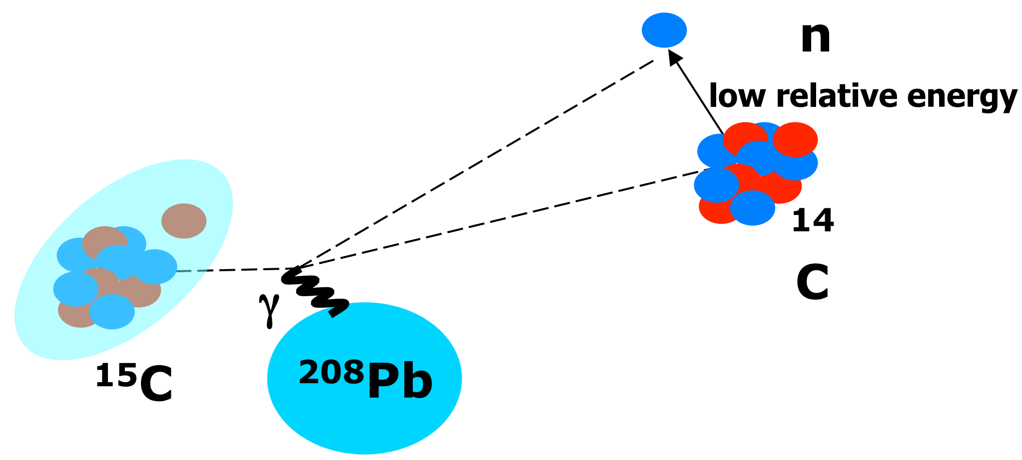
$$\boxed{[\hat{T}_R + V_{pp}(R) - E_p]\psi_p(\mathbf{R}) + \sum_{p'\neq p} V_{pp'}(R)\psi_{p'}(\mathbf{R}) = 0}$$

Coupling potentials: $V_{pp'}(R) = \langle\tilde{\phi}_p(r)|U_{vt}+U_{ct}|\tilde{\phi}_{p'}(r)\rangle$

Energies: $E_p = E - \tilde{\epsilon}_p$ $\quad\tilde{\epsilon}_p = \langle\tilde{\phi}_p(\mathbf{r})|H_{\mathrm{int}}|\tilde{\phi}_p(\mathbf{r})\rangle$

- **direct measurement** $^{14}C(n,\gamma)^{15}C$

- **Coulomb dissociation**



Coulomb dissociation for (n,γ)

CDCC + set of single particle parameters

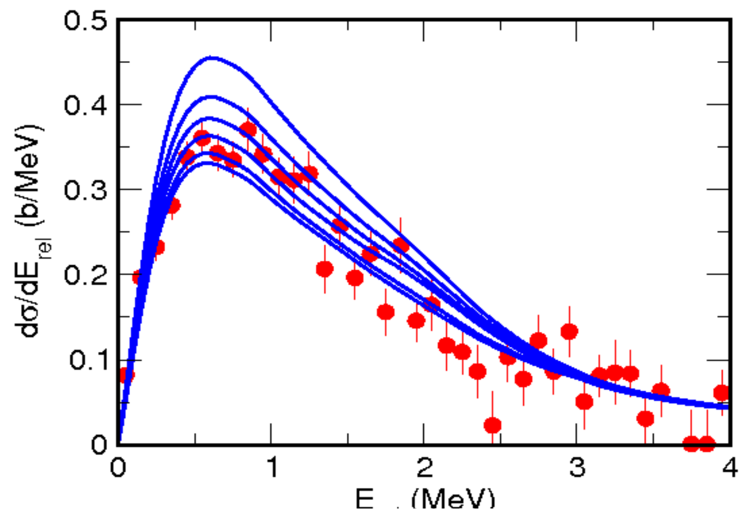
- extract ANC from χ^2 minimum
  - error from $\varepsilon=\chi_{min}^2+1$

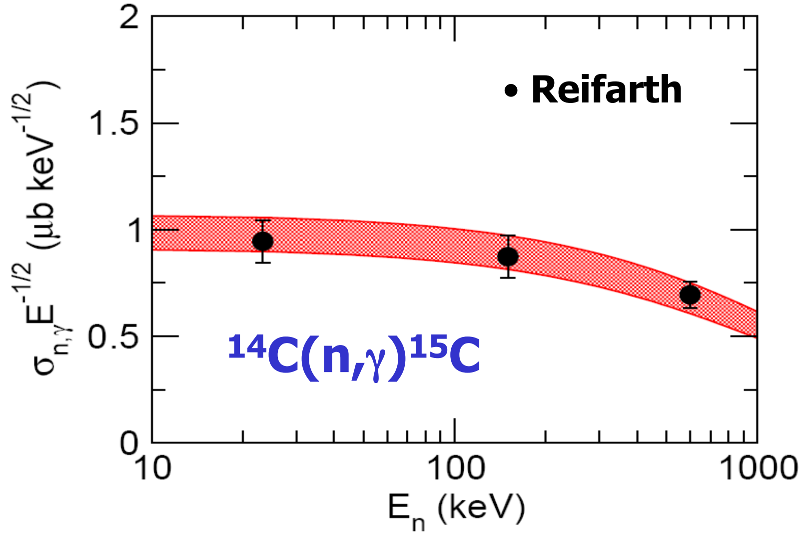
Yao, JPG33 (2006) 1

$$ANC = 1.32 \pm 0.07 \text{ fm}^{-1/2}$$

Summers et al., PRC78(2009)069908

$^{208}Pb(^{15}C,^{14}C+n)^{208}Pb$@68 MeV/u

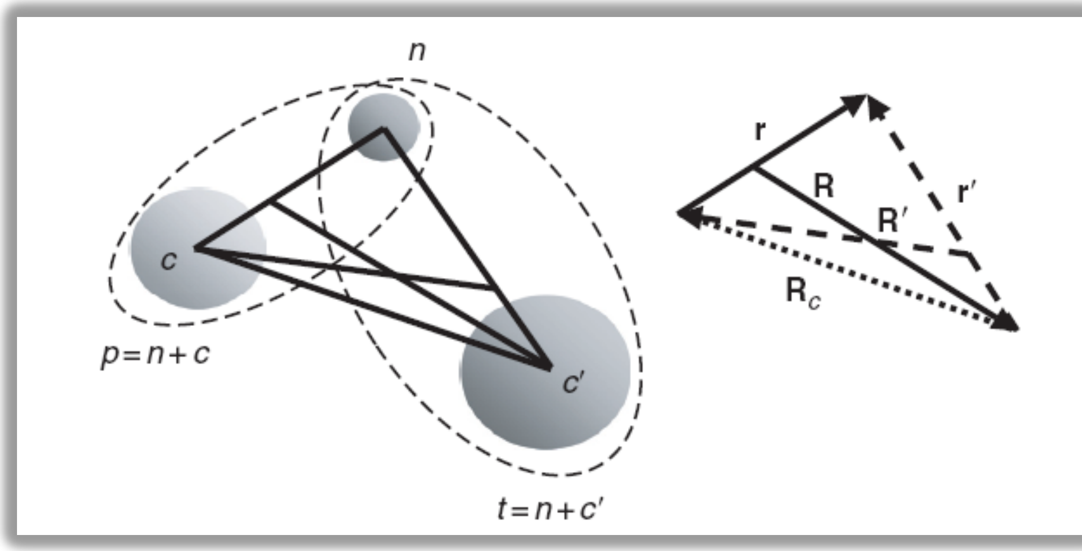




Nakamura et al, NPA722(2003)301c
Reifarth et al, PRC77,015804 (2008)

one nucleon transfer:coordinates

$$\mathbf{r} = p\mathbf{R}' + q\mathbf{R} \text{ and } \mathbf{r}' = p'\mathbf{R}' + q'\mathbf{R}$$



Initial and final bound states:

$$[H_p - \varepsilon_p]\phi_p(\mathbf{r}) = 0 \quad \text{where} \quad H_p = T_\mathbf{r} + V_p(\mathbf{r})$$

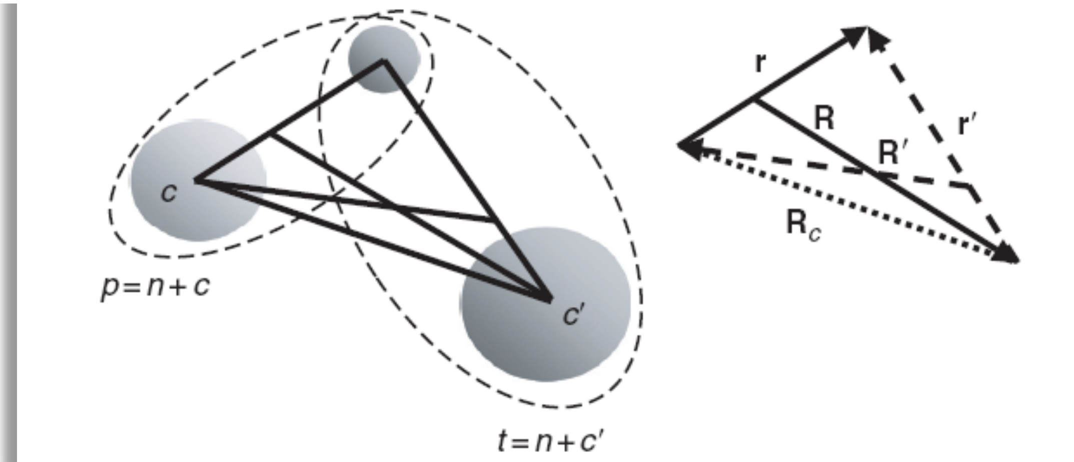
$$[H_t - \varepsilon_t]\phi_t(\mathbf{r}') = 0 \quad \text{where} \quad H_t = T_{\mathbf{r}'} + V_t(\mathbf{r}').$$

$$Q = \varepsilon_p - \varepsilon_t$$

one nucleon transfer: operator

$$H = T_{\mathbf{r}} + T_{\mathbf{R}} + V_p(\mathbf{r}) + V_t(\mathbf{r}') + U_{c'c}(\mathbf{R}_c),$$

$$T_{\mathbf{r}} + T_{\mathbf{R}} = T_{\mathbf{r}'} + T_{\mathbf{R}'}$$



$$\boxed{\begin{aligned} H &= H_{\text{prior}} = T_{\mathbf{R}} + U_i(R) + H_p(\mathbf{r}) + \mathcal{V}_i(\mathbf{R},\mathbf{r}) \\ &= H_{\text{post}} = T_{\mathbf{R}'} + U_f(R') + H_t(\mathbf{r}') + \mathcal{V}_f(\mathbf{R}',\mathbf{r}'), \end{aligned}}$$

$$\mathcal{V}_i(\mathbf{R},\mathbf{r}) = V_t(\mathbf{r}') + U_{c'c}(\mathbf{R}_c) - U_i(R)$$

$$\text{or} \quad \mathcal{V}_f(\mathbf{R}',\mathbf{r}') = V_p(\mathbf{r}) + U_{c'c}(\mathbf{R}_c) - U_f(R').$$

one nucleon transfer: auxiliary potential

$$\mathcal{V}_i(\mathbf{R},\mathbf{r}) = V_t(\mathbf{r}') + U_{c'c}(\mathbf{R}_c) - U_i(R)$$

$$\text{or} \quad \mathcal{V}_f(\mathbf{R}',\mathbf{r}') = V_p(\mathbf{r}) + U_{c'c}(\mathbf{R}_c) - U_f(R').$$

binding potential

remnant term

U_i is an auxiliary potential and therefore the solution is independent of that choice!

- standard choice in DWBA U_i is optical potential reproducing elastic scattering
- standard choice in CDCC U_i is U_{ct} + U_{xt} folded over the bound state c+x
- other possible choise U_i = $U_{cc'}$ ($R_{cc'}$) to cancel the remnant term
- etc…

Features of transfer reactions

Q-value matching: similar k in incident and exit distorted waves

$$\mathbf{T}_{fi}^{\mathrm{DWBA}} = \langle \chi_f^{(-)}(\mathbf{R}_f)\Phi_{I_A:I_B}(\mathbf{r}_f)|\mathcal{V}|\Phi_{I_b:I_a}(\mathbf{r}_i)\chi_i(\mathbf{R}_i)\rangle$$

Angular momentum dependence in zero-range approximation

$$\begin{aligned}\mathbf{T}_{fi}^{\mathrm{PWBA}} &= D_0 \int e^{i\mathbf{q}\cdot\mathbf{R}}\Phi_{I_A:I_B}(\mathbf{R})\mathrm{d}\mathbf{R} \\ &= \sum_{l=0}^{\infty} i^l(2l+1)\int F_l(0,qR)/(qR)P_l(\cos\theta)\Phi_{I_A:I_B}(\mathbf{R})\mathrm{d}\mathbf{R}.\end{aligned}$$

QUIZ: Which line corresponds to larger Q-value?

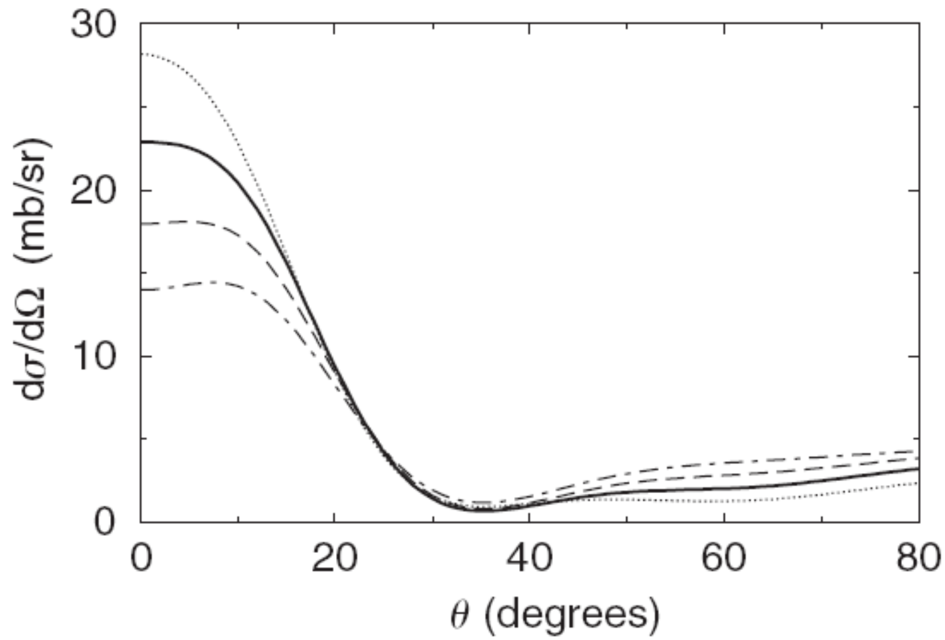
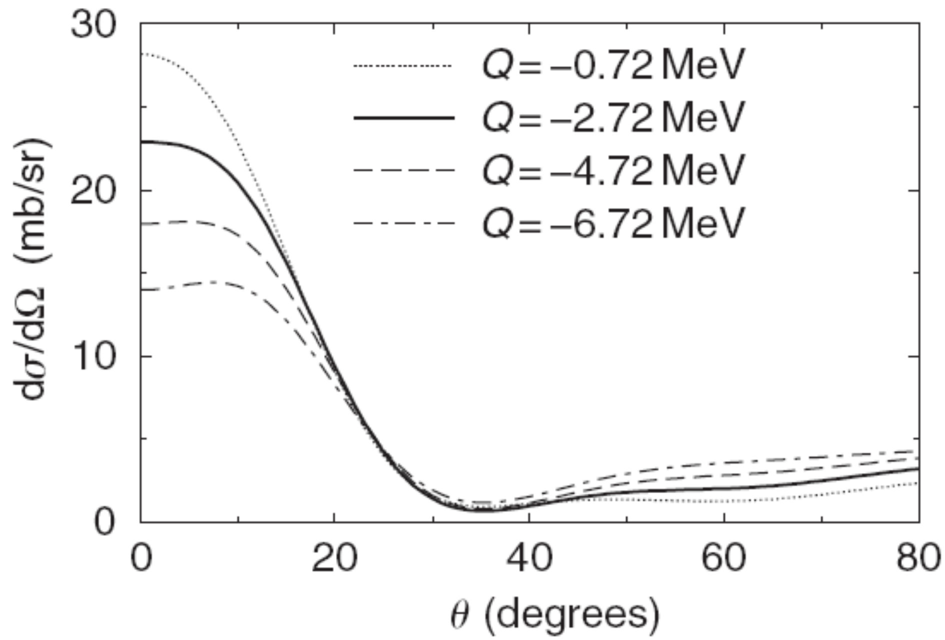


Fig. 14.2. Transfer cross sections for different Q-values for $^{12}C(d,p)^{13}C$ at 20 MeV. The Q-value is varied arbitrarily.

a) Solid
b) Dotted
c) Dashed
d) Dot-dashed

QUIZ: Which line corresponds to larger Q-value?



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a) Solid
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Angular momentum dependence

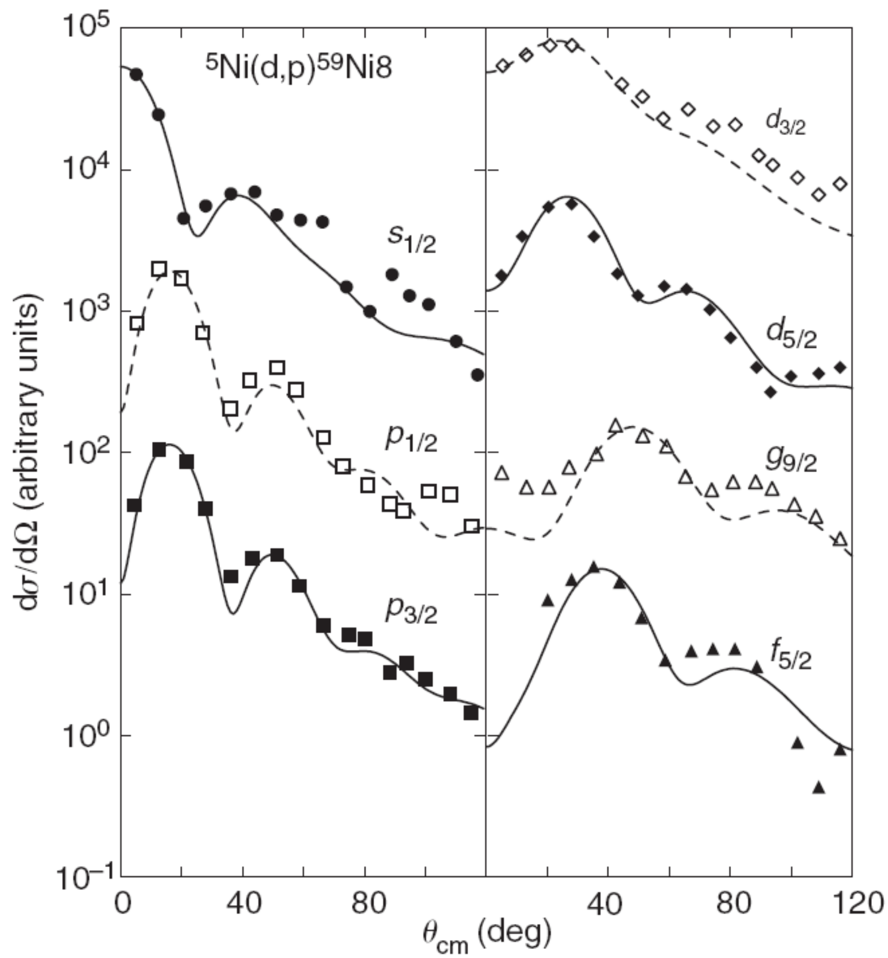


Fig. 14.3. Dependence of the transfer angular distribution on the transferred angular momentum for ^{58}Ni(d,p)^{59}Ni at 8 MeV, with data from [2]. Reprinted from [3], with permission.

ISF whitepaper (2006)

Typical Approximations:

Eikonal

Adiabatic

(d,p) reactions: zero range adiabatic wave approx. (ADWA)

$$\mathbf{T}_{fi} = -\frac{2\mu_f}{\hbar^2 K_f}\langle \psi_f^{(-)}\phi_n|\mathcal{V}_f|\Psi(r,R)\rangle$$

dominating term in $\boldsymbol{\mathcal{V}_f}$ is $V_{np}(r)$, short range
thus exact wfn only needed for small r!

DWBA

$$\phi_0(r)\chi_0(R)$$

ADWA

$$\phi_0(0)\tilde{\chi}(\mathbf{R}) = \Psi^{\mathrm{ad}}(0,\mathbf{R})$$

$$\boxed{(\hat{T}_R + U_{\mathrm{ad}}(R) - (E-\epsilon_0))\tilde{\chi}(\mathbf{R}) = 0,}$$

$$\boxed{U_{\mathrm{ad}}(R) = U_{ct}(R) + U_{vt}(R)}$$

Johnson and Soper potential

(d,p) reactions: finite range adiabatic wave approx

Johnson and Soper potential is based on the zero-range approx

$$U_{\mathrm{ad}}(R) = U_{ct}(R) + U_{vt}(R)$$

Tandy and Johnson reformulated the adiabatic model without the zero-range approx and obtain a new adiabatic potential for the deuteron including breakup effect and finite-range:

$$\boxed{V_{TJ}(R) = \frac{\langle \phi_d \,|\, V_{np}(U_n + U_p) \,|\, \phi_d \rangle}{\langle \phi_d \,|\, V_{np} \,|\, \phi_d \rangle}}$$

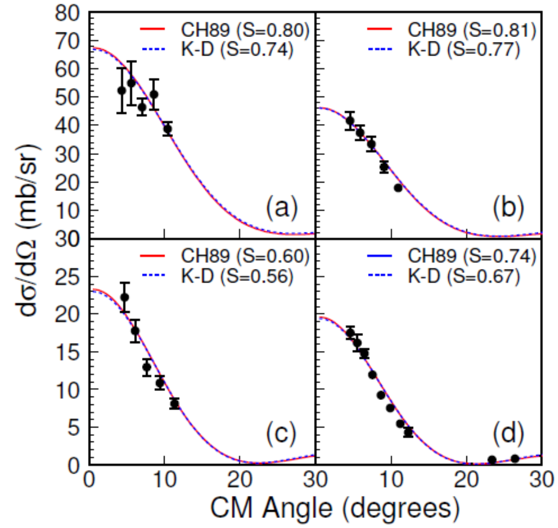
Effective diffuseness increases!

QUIZ: Which of the following sentences is correct?

a) ADWA treats deuteron breakup pertubatively
b) ADWA is valid for high beam energies
c) ADWA takes the np relative energies to be the beam energy
d) ADWA can be used to calculate deuteron elastic scattering

Example of using (d,p) to probe halos

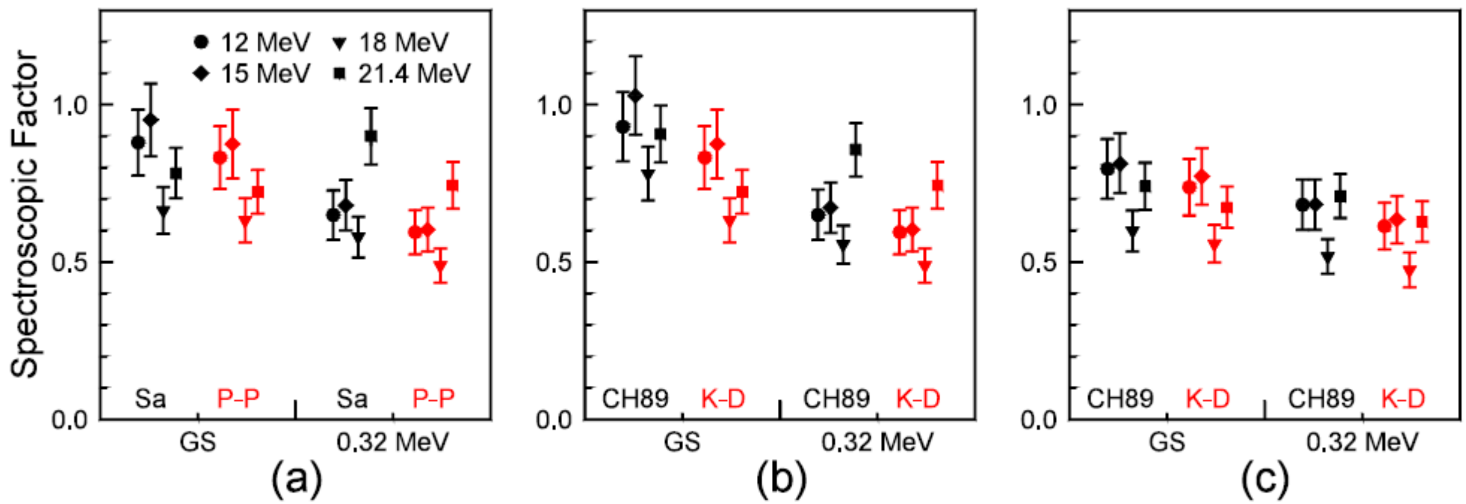
$^{10}Be(d,p)^{11}Be$ @ 12-21 MeV



DWBA
entrance channel

DWBA
exit channel

ADWA

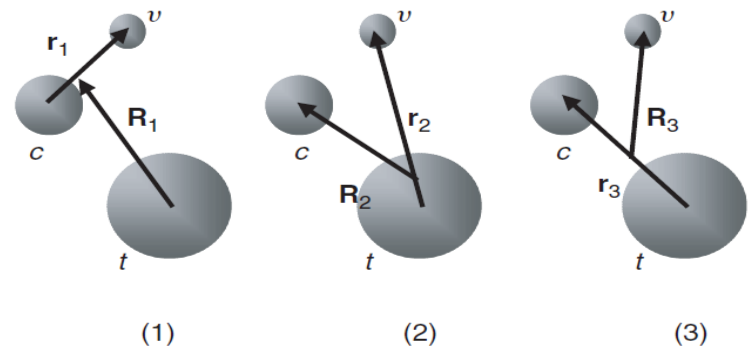


Schmitt et al, PRL 108, 192701 (2012), PRC 88, 064612 (2013)

Testing three-body methods

Faddeev AGS: EXACT
- all three Jacobi components are included
- elastic, breakup and rearrangement channels are fully coupled
- computationally expensive

Deltuva and Fonseca, Phys. Rev. C79, 014606 (2009).

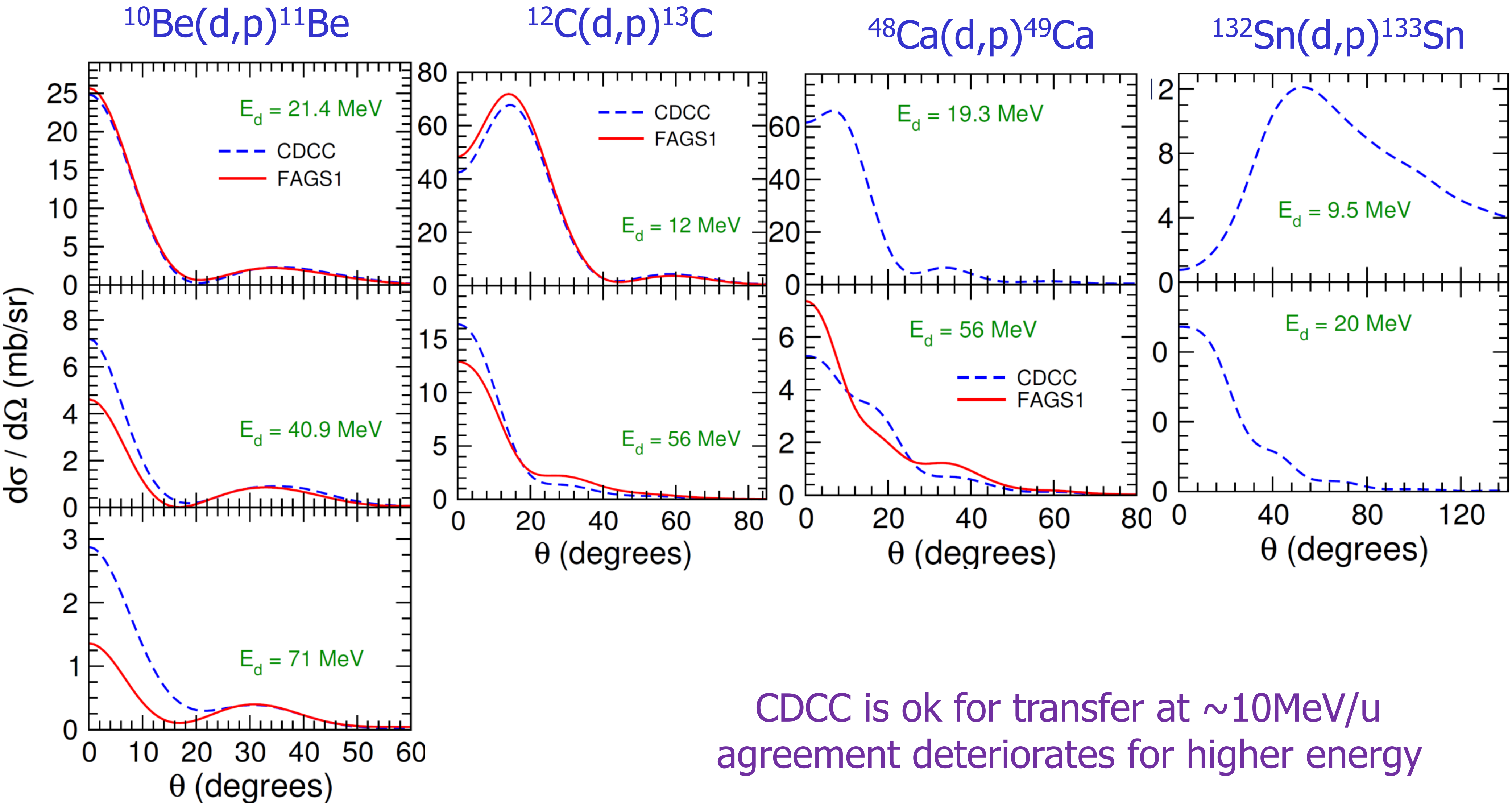


3 jacobi coordinate sets

CDCC:
- only one Jacobi component
- elastic and breakup fully coupled (no rearrangement)
- computationally expensive

ADWA:
- only one Jacobi component
- elastic and breakup fully coupled (no rearrangement)
- approximation for breakup – one term in Weinberg expansion
- runs on desktop – practical use

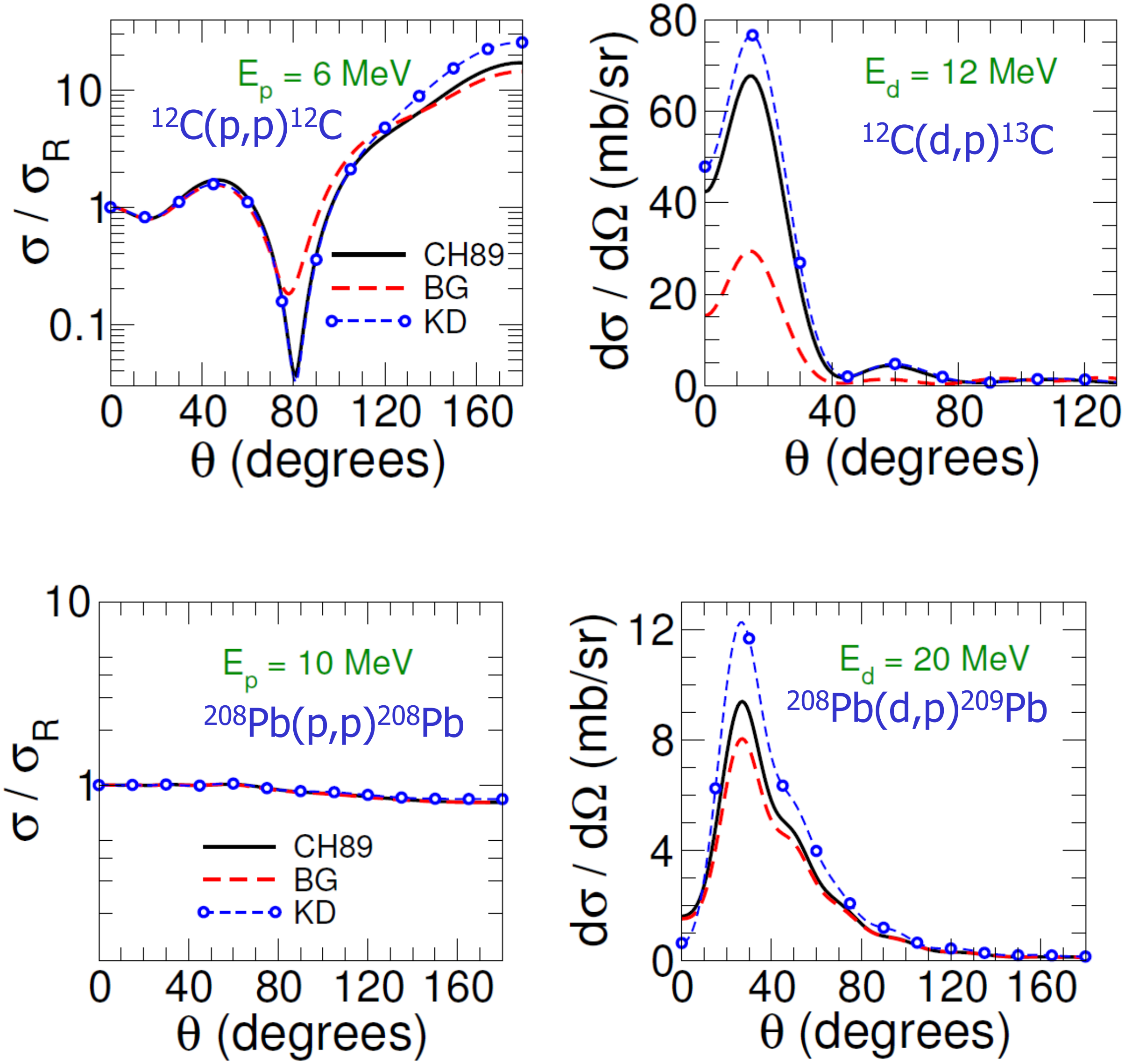
Comparing transfer



CDCC is ok for transfer at ~10MeV/u
agreement deteriorates for higher energy

Upadhyay, Deltuva and Nunes, PRC 85, 054621 (2012)

The dependence on the optical potential



- Constraining p-A elastic reduces uncertainties but remaining uncertainty not neglegible

- Important to include good optical potential information

theory opportunities with FRIB

DOE Nuclear Physics Mission is to understand the fundamental forces and particles of nature as manifested in nuclear matter, and provide the necessary expertise and tools from nuclear science to meet national needs

DOE Nuclear Physics Mission is accomplished by supporting scientists who answer overarching questions in major scientific thrusts of basic nuclear physics research

Science Drivers (Thrusts) from NRC RISAC

Nuclear Structure	Nuclear Astrophysics	Tests of Fundamental Symmetries	Applications of Isotopes
Overarching Questions from NSAC 2007 LRP			
What is the nature of the nuclear force that binds protons and neutrons into stable nuclei and rare isotopes? What is the origin of simple patterns in complex nuclei?	What is the nature of neutron stars and dense nuclear matter? What is the origin of the elements in the cosmos? What are the nuclear reactions that drive stars and stellar explosions?	Why is there now more matter than antimatter in the universe?	What are new applications of isotopes to meet the needs of society?

Overarching questions are answered by rare isotope research

17 Benchmarks from NSAC RIB TF measure capability to perform rare isotope research			
1. Shell structure 2. Superheavies 3. Skins 4. Pairing 5. Symmetries 13. Limits of stability 14. Weakly bound nuclei 15. Mass surface	6. Equation of State (EOS) 7. r-Process 8. $^{15}O(\alpha,\gamma)$ 9. ^{59}Fe supernovae 15. Mass surface 16. rp-Process 17. Weak interactions	12. Atomic electric dipole moment	10. Medical 11. Stewardship

FRIB-CDR, 2010

theory opportunities with FRIB

Reactions	Ab-initio reaction theory, consistent with nuclear structure, adequate for many domains of experimental interest, including (d,p), dripline nuclei and superheavy synthesis; Multi-nucleon transfer, knockout and breakup models for production of nuclei at and beyond the dripline to extract structural information; Unified treatment of structure and reactions for open nuclear systems;	Q1, Q2, Q4: B1-6, B11, B13-15
	Nuclear reactions study at the limits of stability to extract crucial isovector indicators such as neutron skins; Reaction theory with quantified uncertainties for charge-exchange;	Q2, Q3: B3, B17
	Reaction observables to isolate the role of pairing correlations and characterize the pairing interaction;	Q2: B4, B5
	Microscopic theory of spontaneous and neutron-induced fission; Ab-initio theory for light-ion fusion;	Q4: B10,B11
	Reaction theory for fusion consistent with structure	Q2: B2,B14,
	Consistent reaction theory for (d,p) transfer and (n,γ), (p, γ) capture reactions on medium mass and heavy nuclei; Microscopic theory for nuclear fusion to predict thermo- and pycno-nuclear fusion rates in the neutron star crust;	Q1, Q2: B1-6, B16
	Reliable transport theory with quantified errors for heavy-ion reactions from low to intermediate energies;	Q1, Q2: B3,B5,B6