Nuclear **R**eactions – **E**xperiment **II**

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Exotic Beam Summer School – 2014 ORNL

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Test of Sum Rules in Nucleon Transfer Reactions

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Valence nucleon populations in the Ni isotopes

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Application of transfer reactions

- Goal: Test the Macfarlane & French transfer sum rules

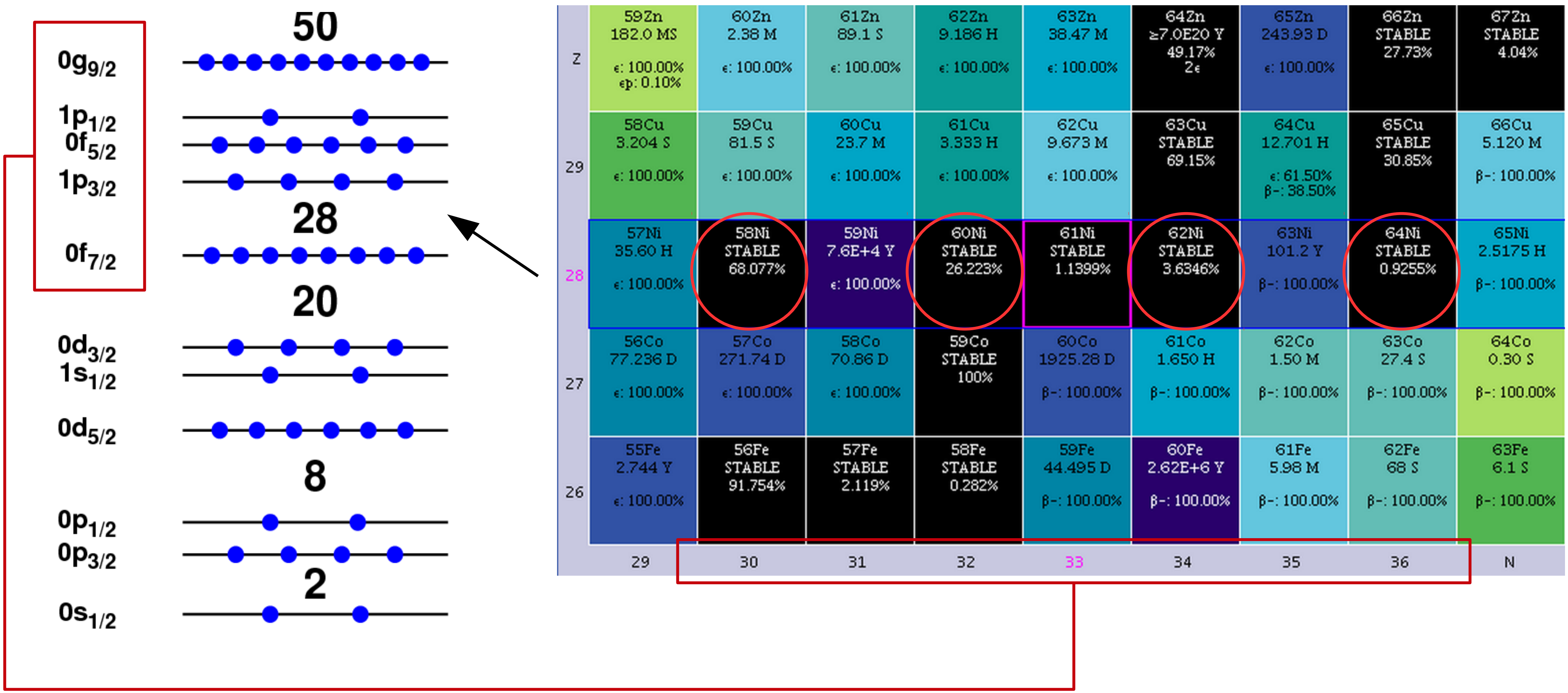
$$N \equiv \frac{1}{(2j+1)}[\Sigma(2j+1)C^2 S_{\text{adding}} + \Sigma C^2 S_{\text{removing}}]$$

- Measure reactions on 4 stable Ni isotopes (^{58,60,62,64}Ni)
- Measure cross sections and check orbital angular momentum assignments
- Extract reduced cross sections
- Normalize to orbital degeneracies
- Investigate robustness of results
  - Deduce orbital occupancies and single-particle energies

How to best test the sum rules?

- Obtain systematic and clear data under near identical experimental conditions
  - Large number of stable isotopes w/ robust targets
  - Well known spin-parity assignments
- Choose reactions best matched momentum for the transfer
  - Requisite particles and beam energies
- Measure large fraction of single-particle strength
  - Sum rules assume full strength identified
- Consistent calculations used for reduction cross sections (Spectroscopic factors)
- Is there are common normalization factor for “all” nuclei?

Active Neutron Orbitals to be Probed



Question

- Which of the following reactions is best suited to study the single-neutron states in these Ni isotopes

A) (^{6}Li,d)

B) (d,p)

C) (d,d')

D) (^{3}He,t)

E) (^{14}C,^{10}C)

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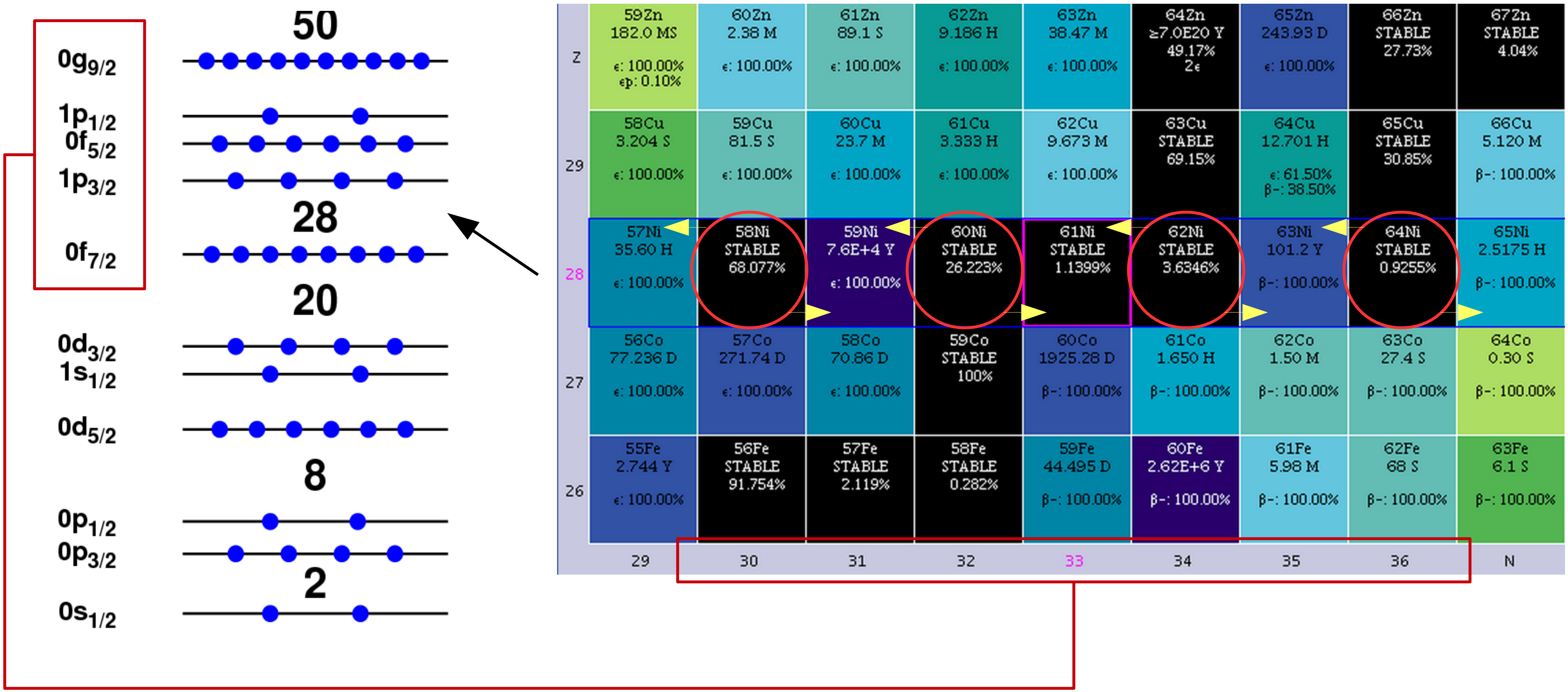
E) (^{14}C,^{10}C)

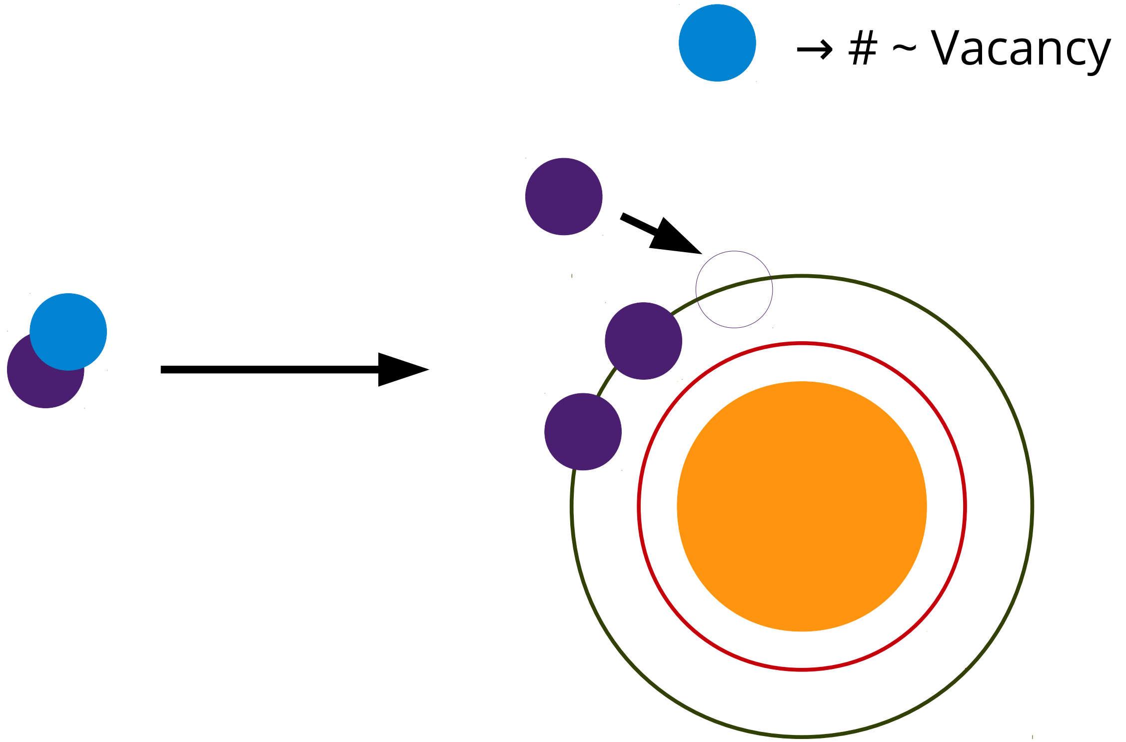
Answer: B) single-neutron adding and removing reactions

Active Neutron Orbitals to be Probed



Choice of Reaction Type

- Single-neutron adding probes vacancies (# of holes)
- Single-neutron removal probes occupancies (# of particles)
- Low energy elastic scattering provides reliable absolute cross sections



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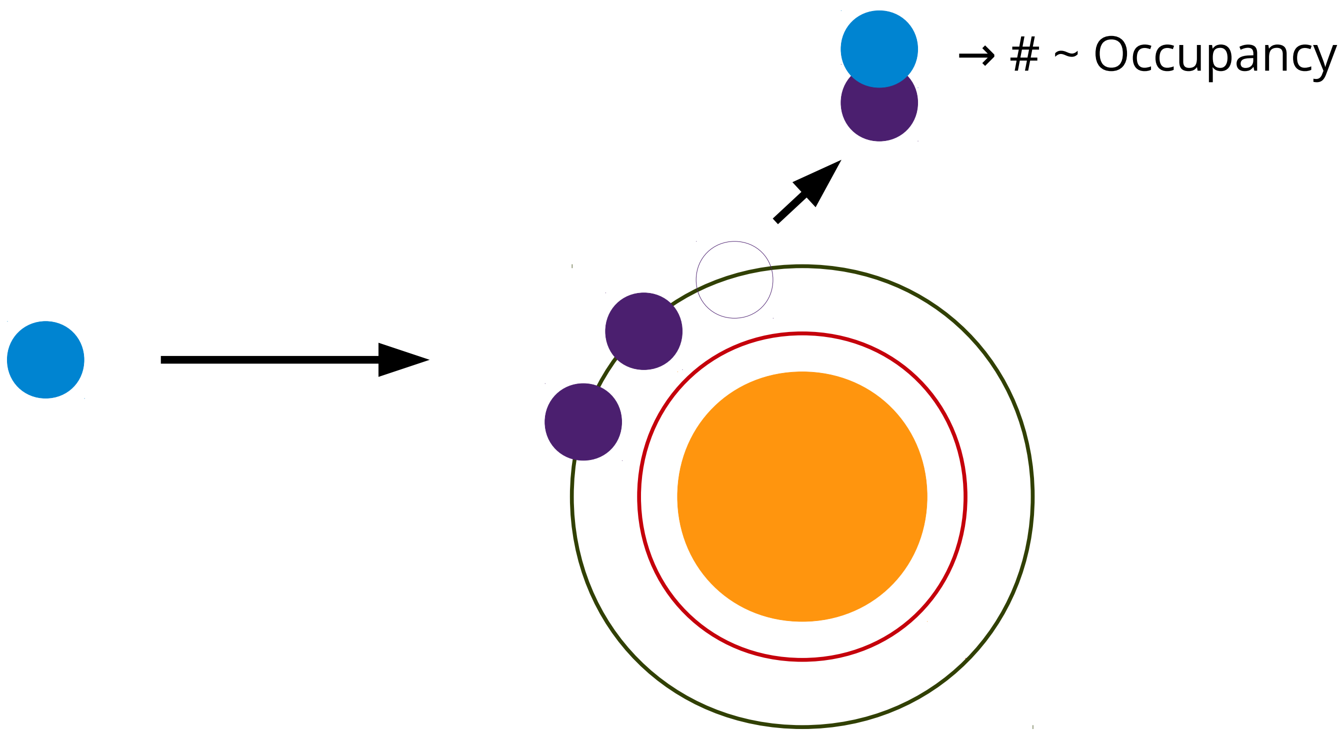

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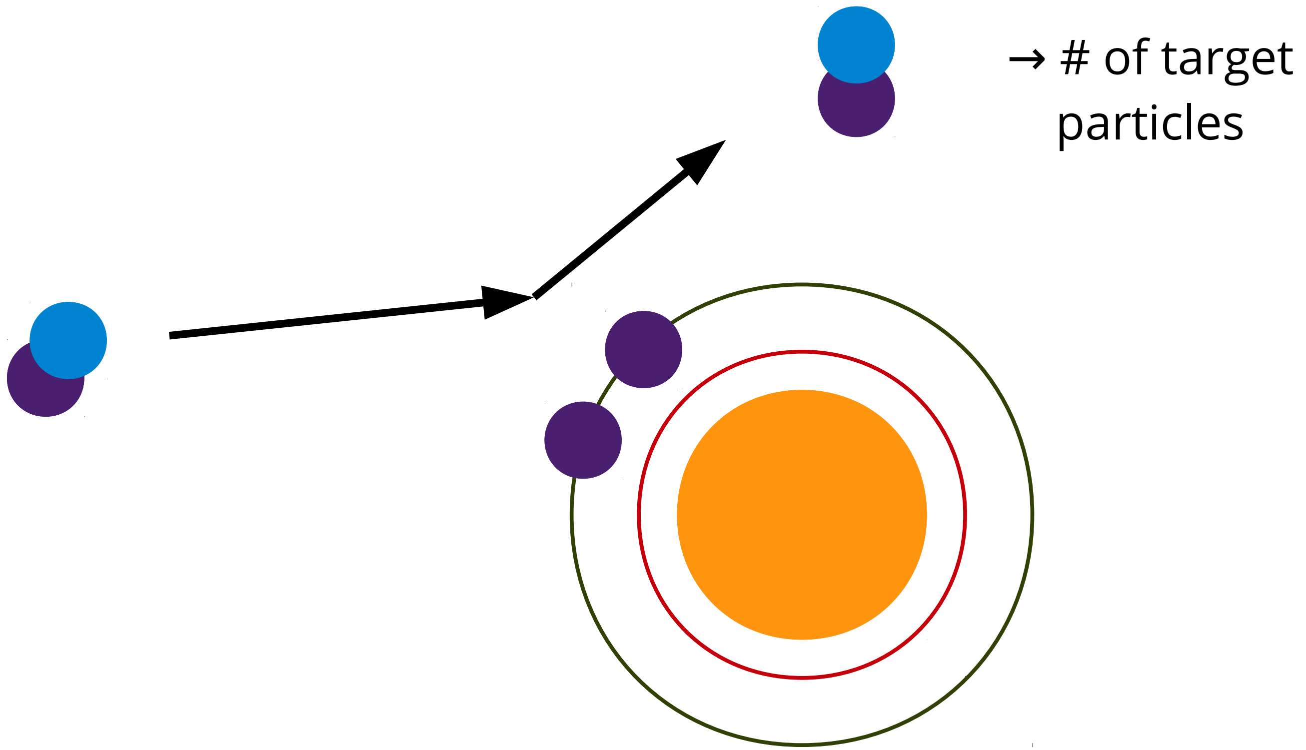

Question

- Which reaction is best matched for ***l* = 4 neutron** transfer on ^{64}Ni?

A) (d,p) Q = +3.9 MeV

B) (^{3}He,t) Q = -1.7 MeV

C) (^{4}He,^{3}He) Q = -14.5 MeV

D) (^{4}He,t) Q = -12.4 MeV

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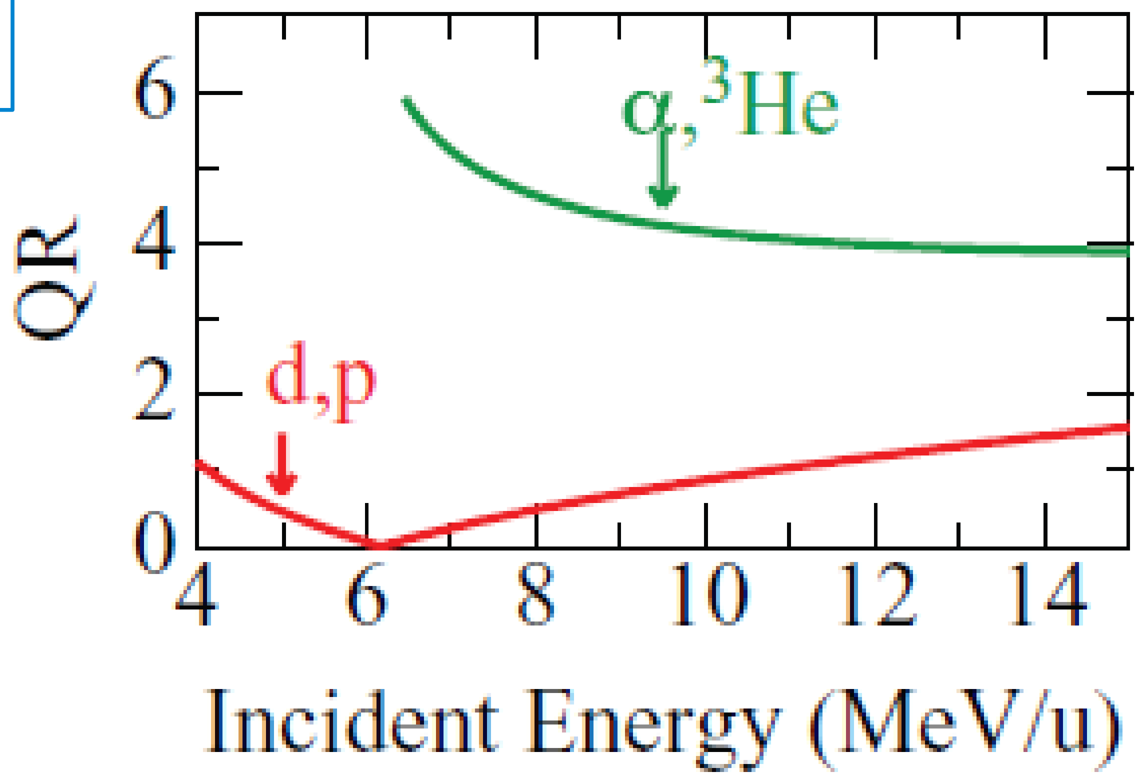
Answer: C) (^{4}He,^{3}He) Q = -14.5 MeV

Choice of Reaction Type

Momentum matching conditions

- Strong Q value dependence, along with beam energy and radial dependence

$$\ell \leq qR$$



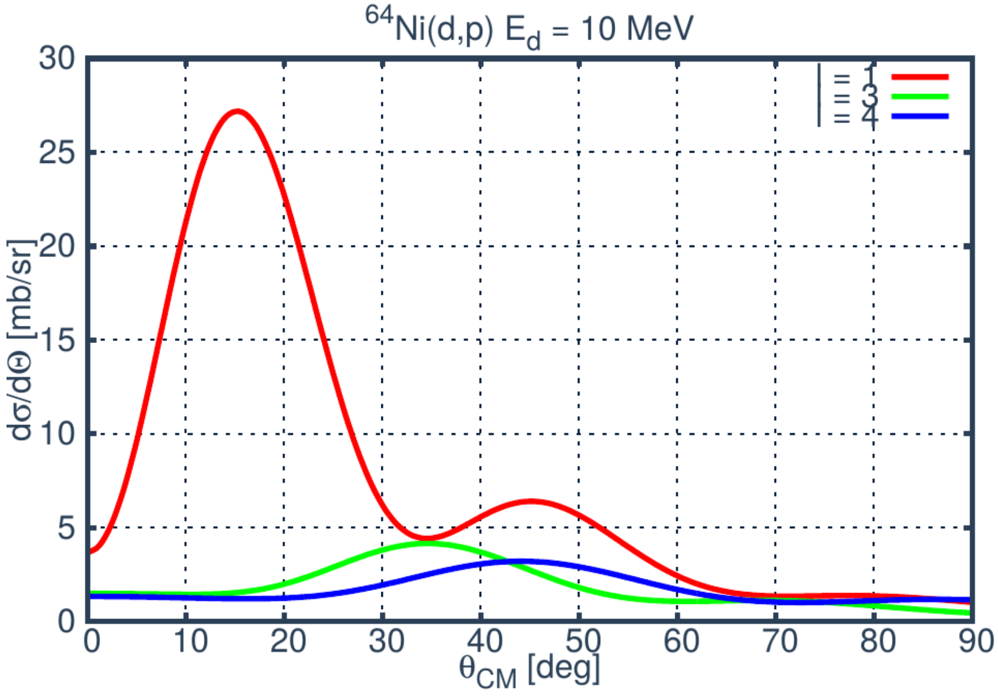
$l = 3 \rightarrow 0f_{5/2}$

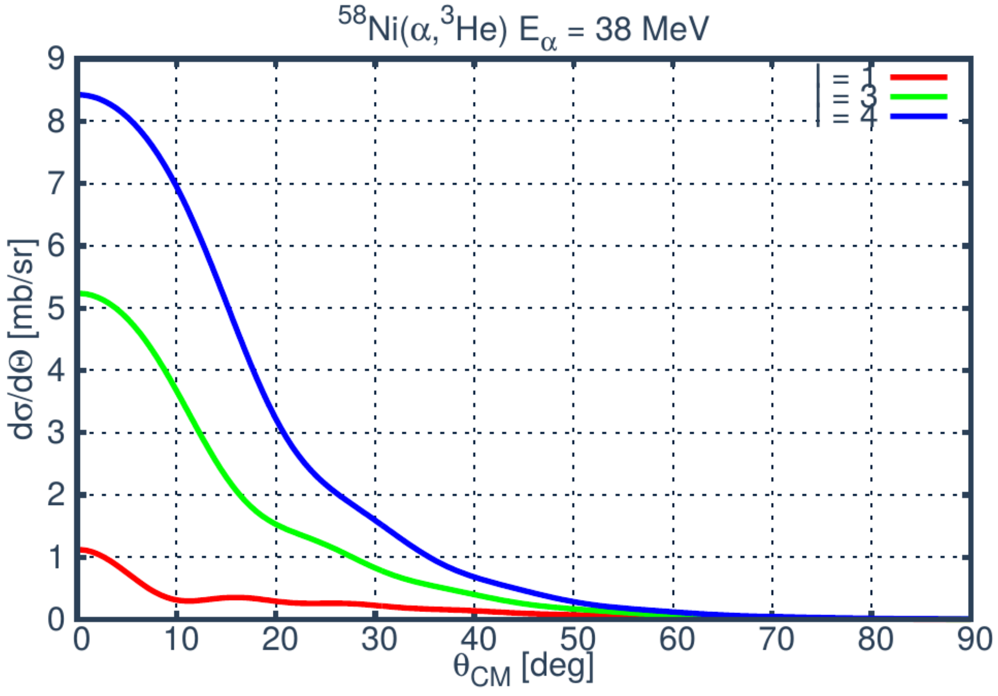
$l = 1 \rightarrow 1p_{3/2(1/2)}$

Estimated Cross Sections & Distributions

- Calculated differential cross sections from DWBA with global parameter sets [e.g., An & Cai, PRC 73 (2006), Becchetti & Greenlees PR (1969)]
- Angle dependencies and momentum matching

***l* = 1 → 1$p_{3/2(1/2)}$ *l* = 3 → 0$f_{5/2}$ l = 4 → 0$g_{7/2}$**





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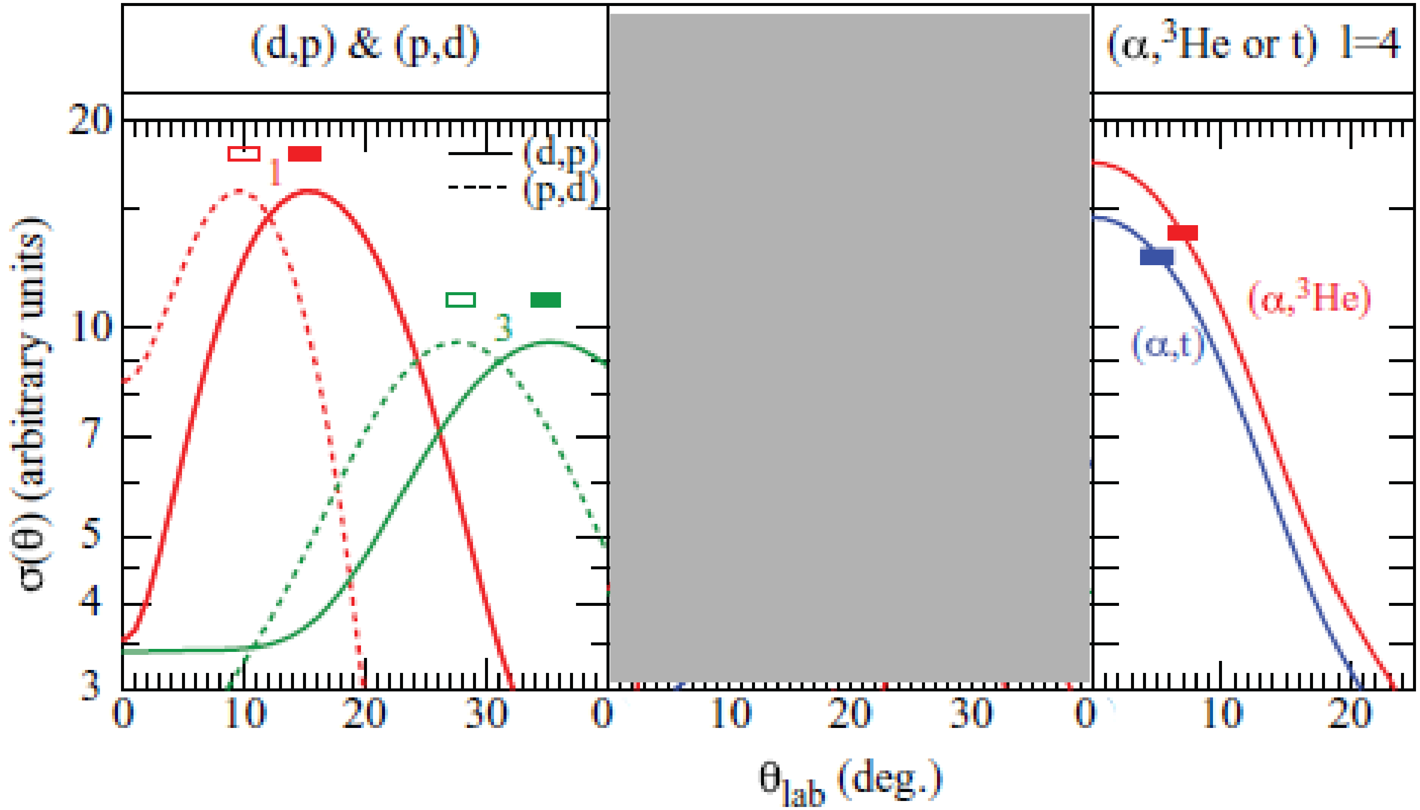
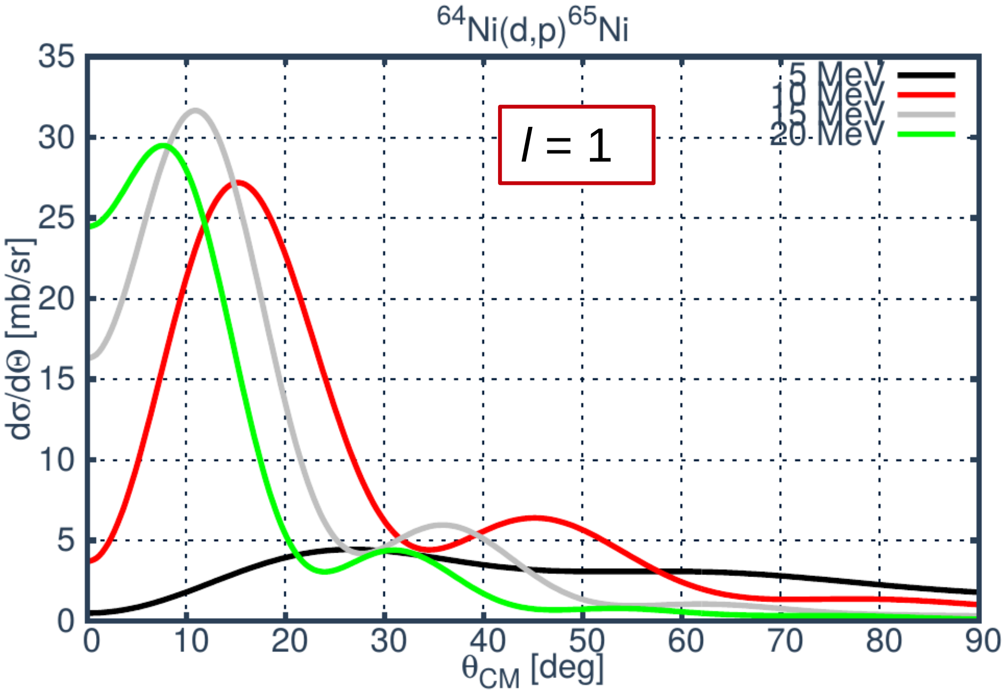
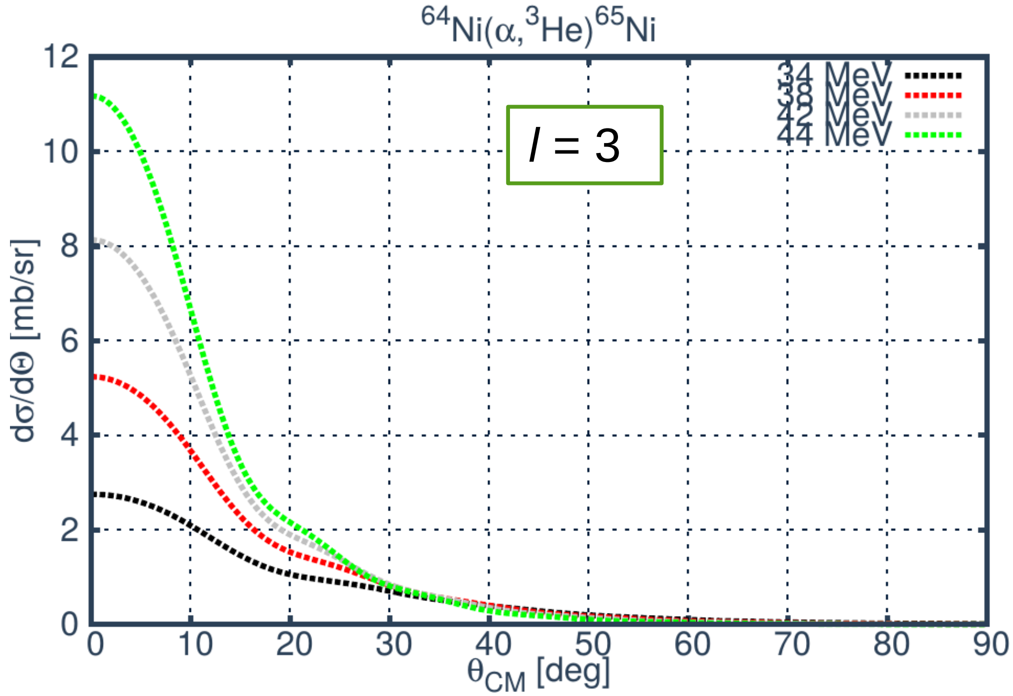


Estimated Cross Sections & Distributions

- Calculated differential cross sections from DWBA with global parameter sets [e.g., An & Cai, PRC 73 (2006), Becchetti & Greenlees PR (1969)]
- Cross section and angular distribution sensitivity to incoming beam energy





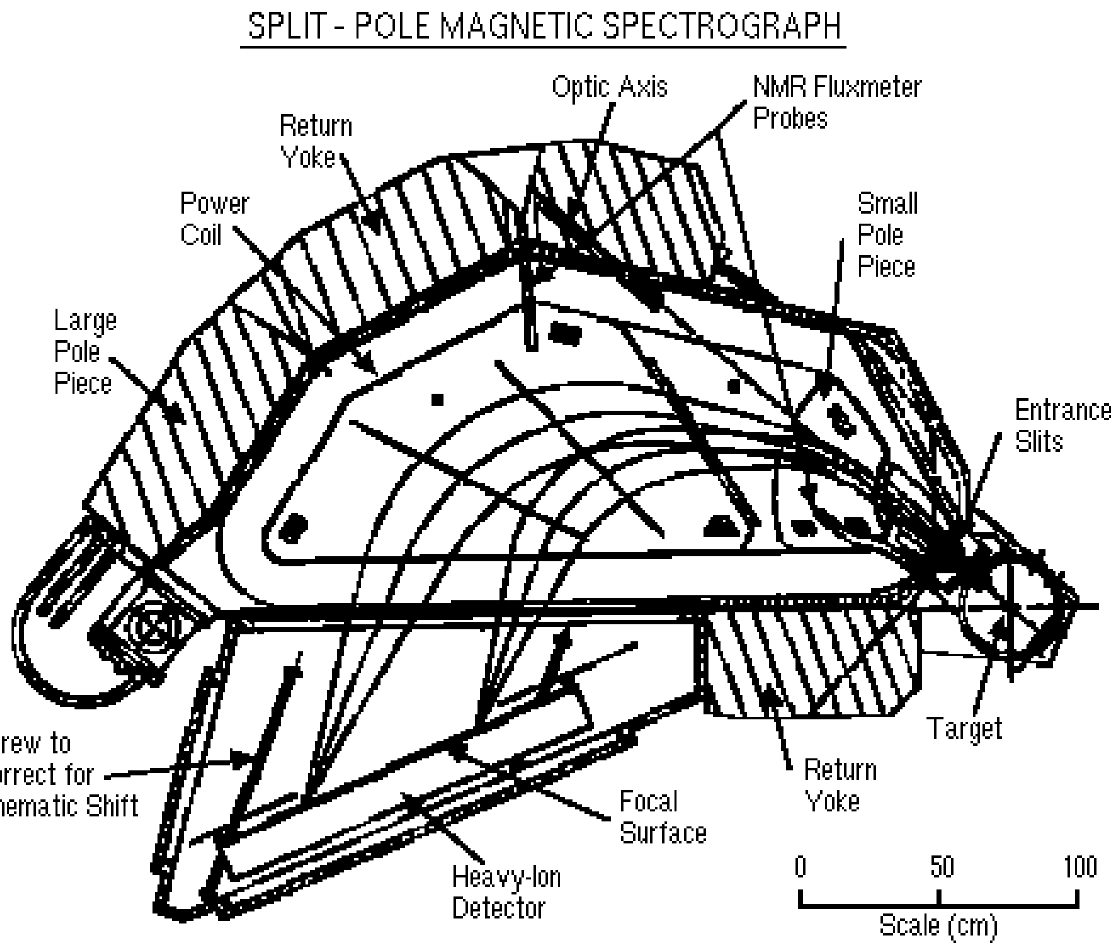
Summary of Reaction Conditions

Reaction	Beam energy (MeV)	θ_{LAB} (deg)	FWHM (keV)
(d,p)	10	15	33
		35	
(p,d)	28	10	48
		25	
$(\alpha,^3He)$	38	7	50
$(^3He,\alpha)$	25	5	75

- Measure each reaction on each target
  - x4 targets, x4 reactions, x6 angles → a lot of measurements
  - Completed in ~5 days of beam time

Experimental setup and data

- Yale ESTU tandem and Enge split-pole spectrograph



Dimensions of the Split-Pole Spectrograph are shown in the drawing where trajectories of particles with two different Bρ's are indicated

Target Thicknesses and Cross Sections

α scattering in Rutherford regime

$$\frac{d\sigma}{d\Omega_R}(\theta_R) = \frac{Y_R}{n_R \cdot t_R \cdot \Omega_R}$$

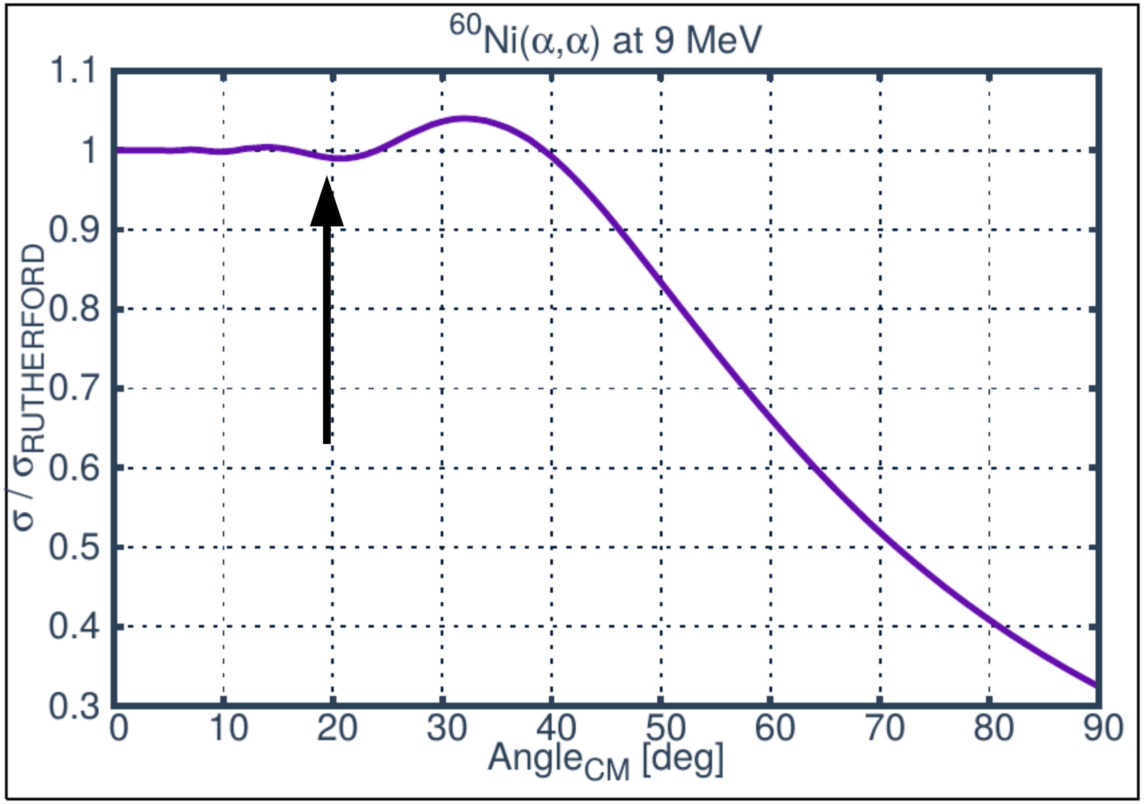
Y_R → # of observed particles

n_R → # of beam particles

t_R → # target particles

Ω_R → Detector solid angle

$$t_R = \frac{Y_R}{n_R \cdot \frac{d\sigma}{d\Omega_R}(\theta_R) \cdot \Omega_R}$$



Nucleus	Thickness (μg/cm^2)	Purity (%)
^{58}Ni	211	99.6
^{60}Ni	204	99.7
^{62}Ni	219	96.5
^{64}Ni	160	91.0

Raw counts to cross sections

- Key to have reliable absolute cross sections with small systematic and statistical uncertainties
- Same target position and aperture as Rutherford scattering data

$$\frac{d\sigma}{d\Omega}(\theta) = \frac{Y}{n \cdot t \cdot \Omega}$$

Y → # of observed particles

n → # of beam particles

t → # target particles

Ω → Detector solid angle

$$\frac{\frac{d\sigma}{d\Omega}(\theta)}{\frac{d\sigma_R}{d\Omega_R}(\theta_R)} = \frac{Y}{Y_R} \cdot \frac{n_R}{n} \cdot \frac{t_R}{t} \cdot \frac{\Omega_R}{\Omega}$$

$\frac{t_R}{t} \cdot \frac{\Omega_R}{\Omega}$ =1

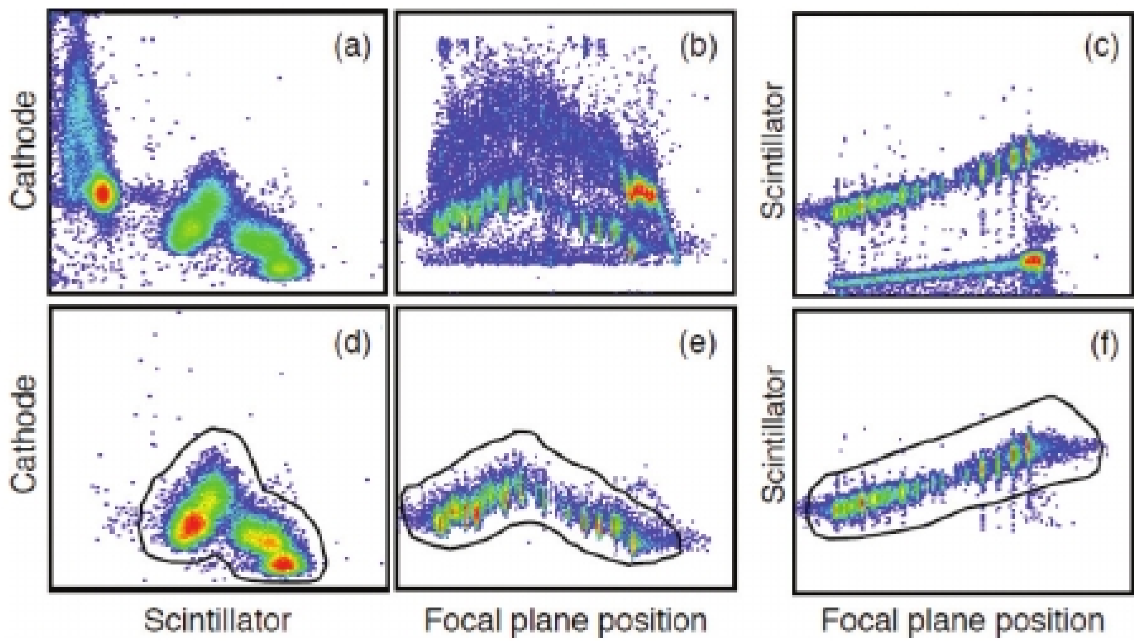
$$\frac{d\sigma}{d\Omega}(\theta) = \frac{\frac{d\sigma}{d\Omega_R}(\theta_R) \cdot n_R}{Y_R} \cdot \frac{Y}{n}$$

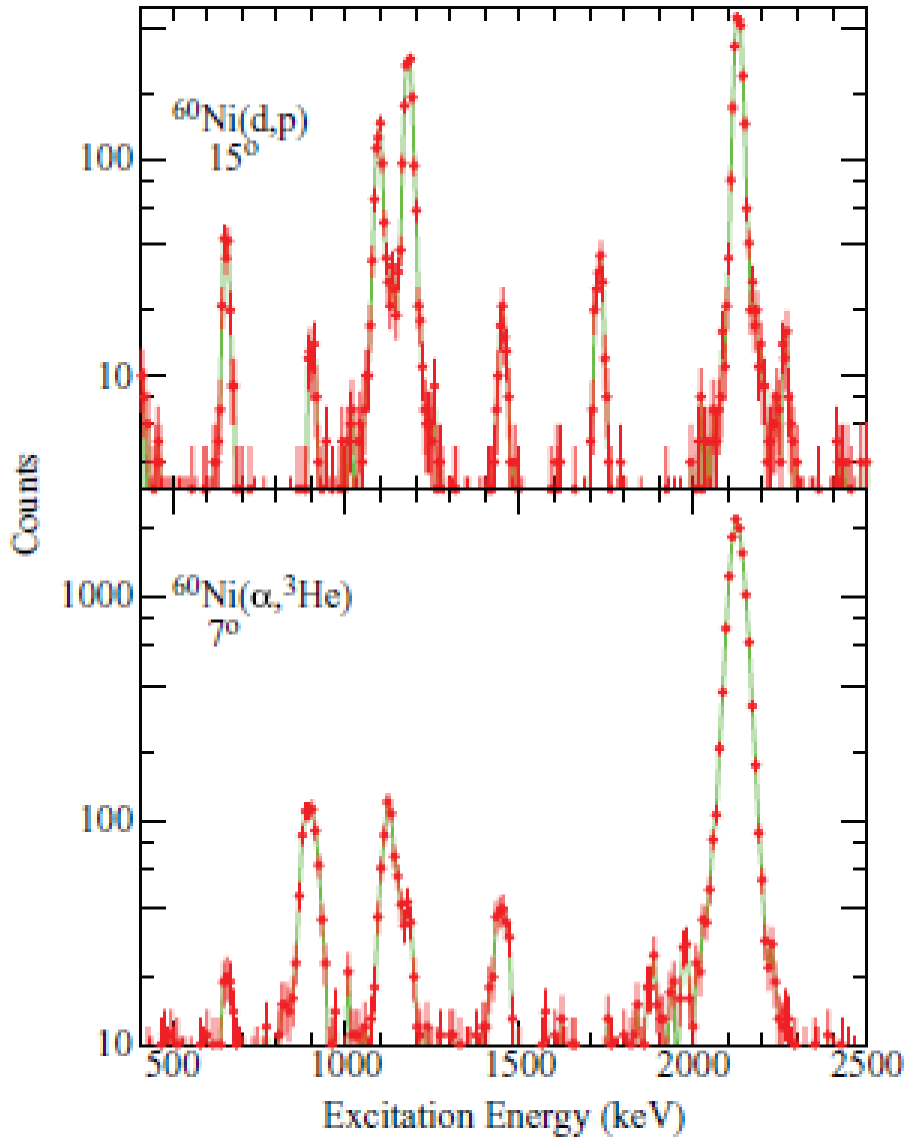
$$\frac{d\sigma}{d\Omega}(\theta) = F \cdot \frac{Y}{n}$$

Absolute cross sections within ~5%

Examples of data taken

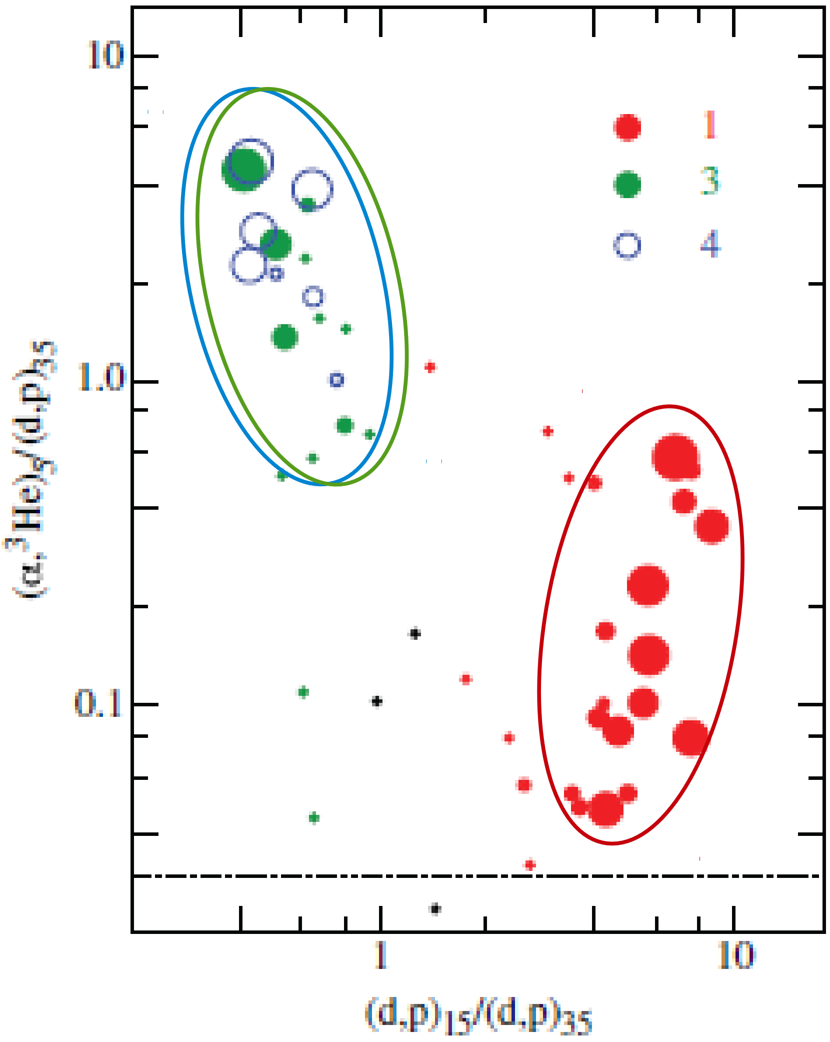
^{60}Ni Target: Identification of the protons in (d,p)



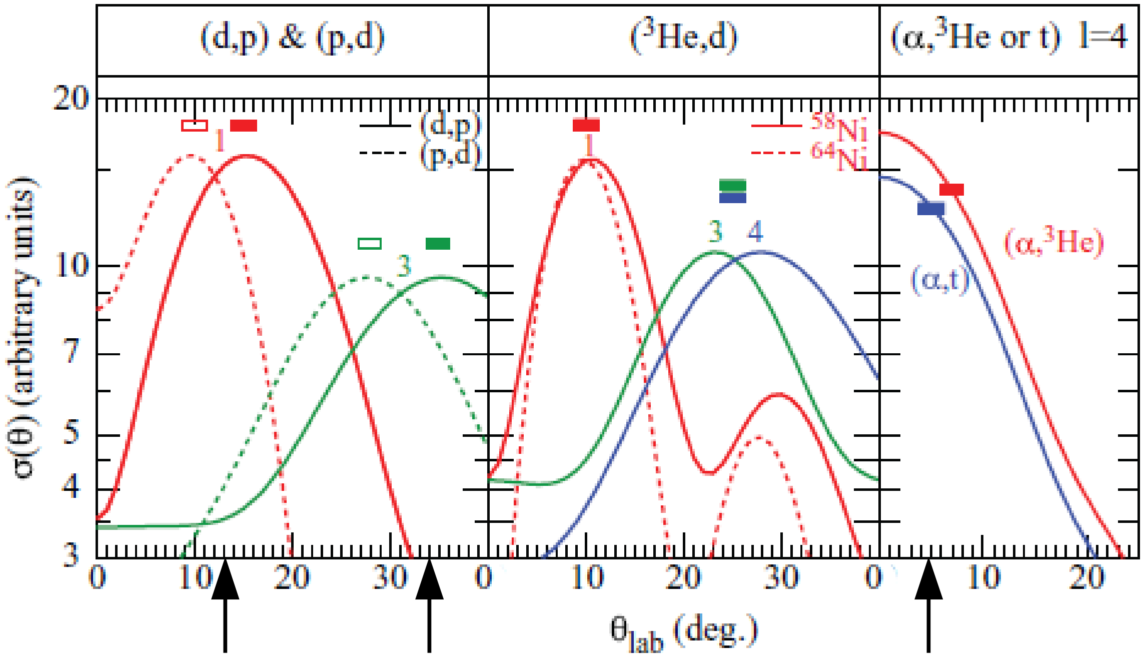


Well known excitation energies; small backgrounds: ~10% or less

Confirmation of / Assignments



- Did not measure full ang. dist.
- Check though by looking at ratios of measure cross sections
- Dot size proportional to cross section



Reduced cross sections: Relative Spectroscopic Factors

- Normalize the calculated cross sections to measured cross sections
  - Distorted wave Born approximation (DWBA) calculations (~10% variations between parameter sets)
- Global potentials: Parametrized to data as function of Beam E, Target Z, & A

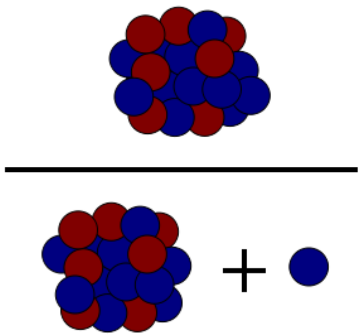
Statistical factor

Calculated cross section for "pure" single-particle like state

$$\left.\frac{d\sigma}{d\Omega}\right|_{Measured} = g S_{lj} \left.\frac{d\sigma}{d\Omega}\right|_{DWBA}$$

Amount of overlap between initial and final states
Spectroscopic Factor

S_{lj} =

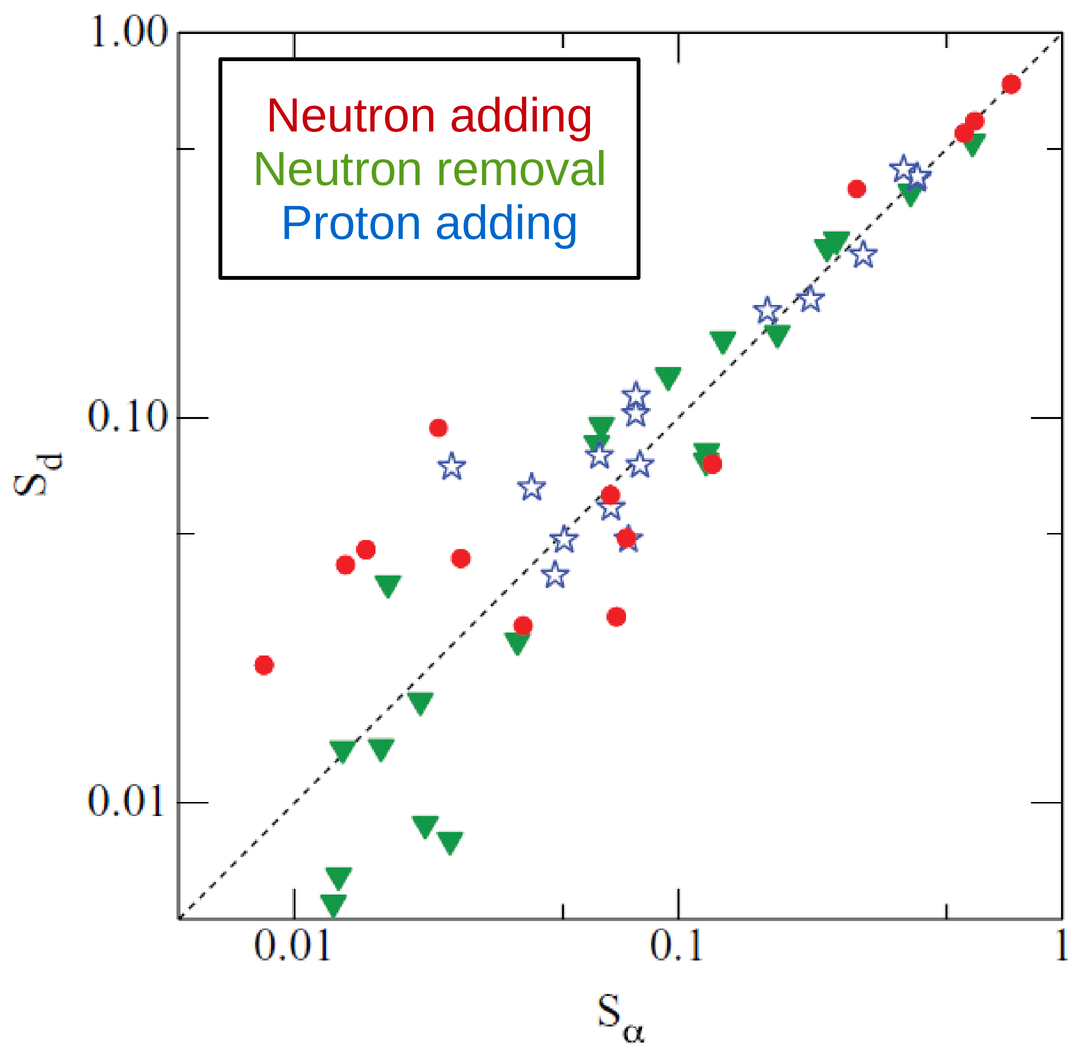


Final state

Initial state + nucleon

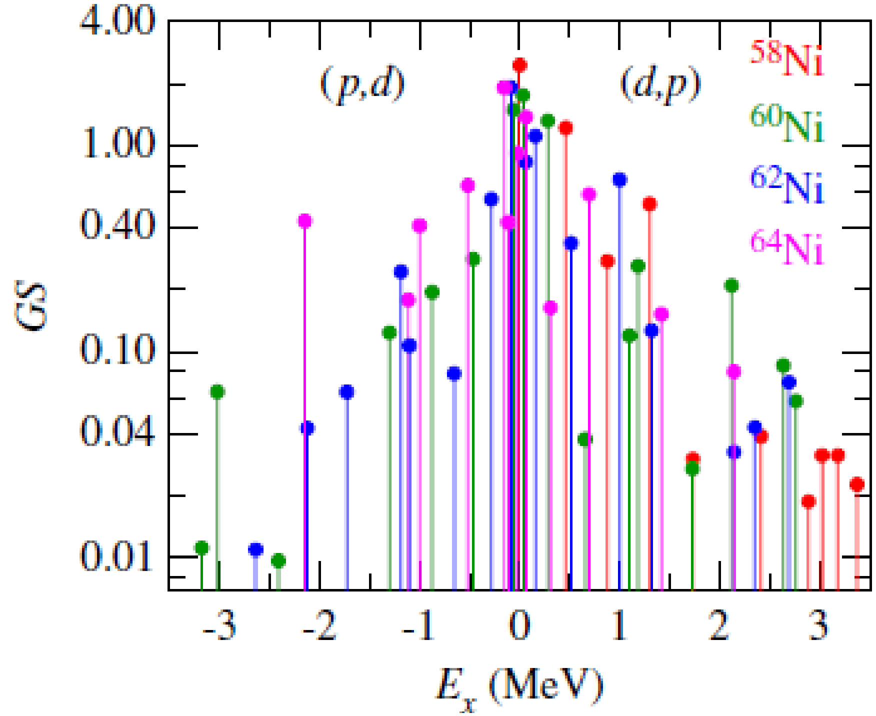
Consistency of Reduced cross sections

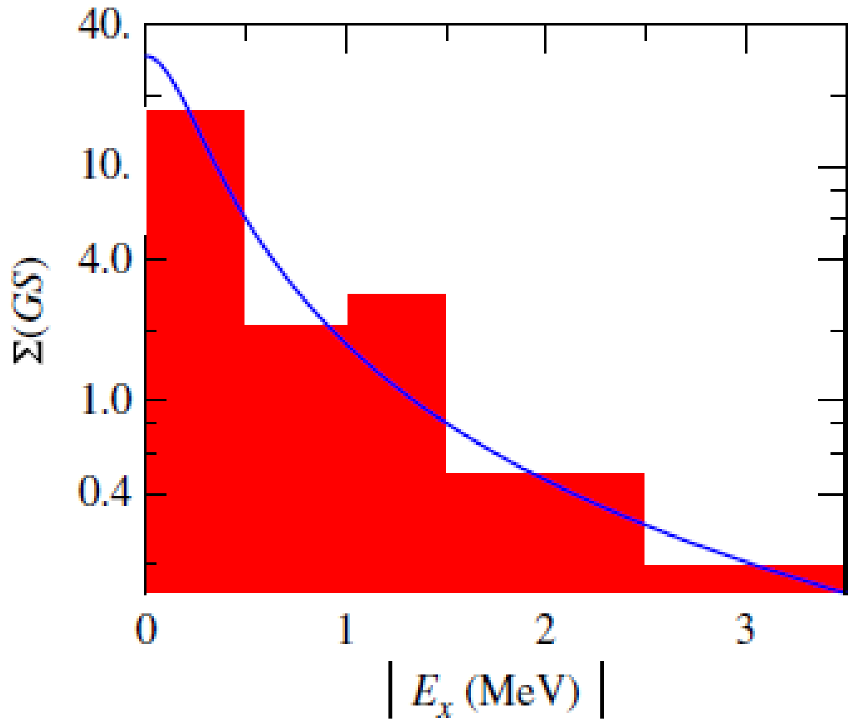
- Spectroscopic factors (S) for *l* = 3 states
- Deduced from both reactions from *Z* = 1 & 2 projectiles
- e.g., (d,p) & (^{4}He,^{3}He)
- Small deviations for states with maximum strength (S > ~0.1)



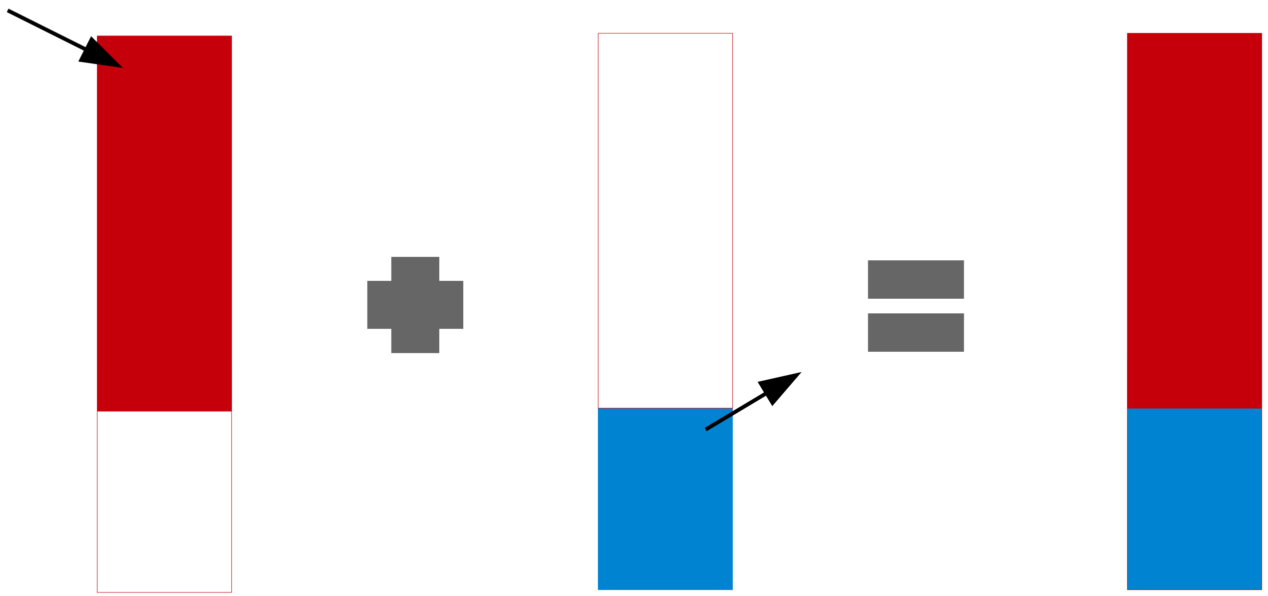
Was a majority of the strength measured?

- Strength (GS) → $(2j+1)C^2S$ for neutron adding, C^2S for removal
- E_x → Energy of (l = 1) state relative to binding of target ground state
- Fits suggest < 3-5% of strength lies above E_x = 3.5 MeV





Extract normalization value using sum rules for each target



Orbital (njl)

Vacant

Occupied

Neutron Adding probes vacancy [# of holes]

Neutron Removal probes occupation [# of particles]

Degeneracy:

V + **O** = 2j + 1

[$1p_{3/2}$ = 4]

Macfarlane & French SUM RULE

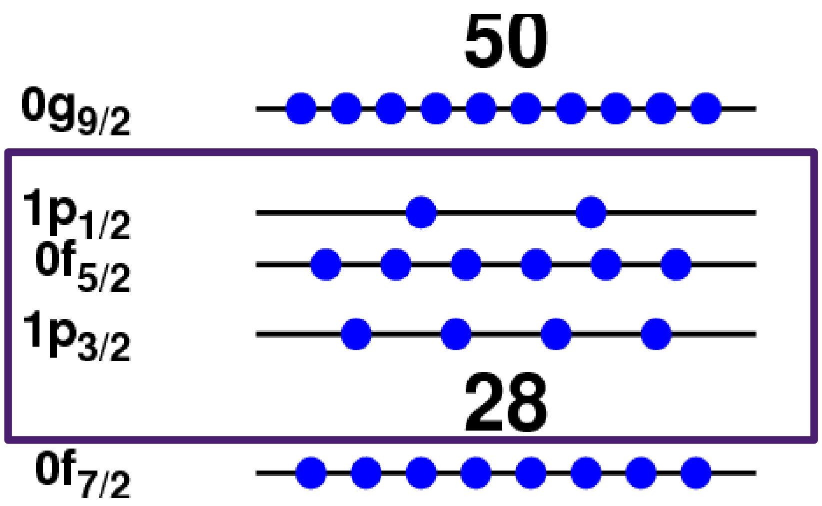
$$N \equiv \frac{1}{(2j+1)}[\Sigma(2j+1)C^2 S_{\text{adding}} + \Sigma C^2 S_{\text{removing}}]$$

Normalization [for single orbital]

Macfarlane and French, Rev. Mod. Phys. 32, 567 (1960)

Normalize the S values for each Ni isotope

- Three different ways to normalize
  - l = 1 from (d,p)/(p,d)
  - l = 3 from (d,p)/(p,d)
  - l = 3 from (^{4}HE,^{3}He)/(^{3}He,^{4}He)



- Sum $1p_{3/2}$ + $1p_{1/2}$ = 6 → l = 1
- Degeneracy $0f_{5/2}$ = 6 → l = 3

Macfarlane & French SUM RULE

$$N \equiv \frac{1}{(2j+1)}[\Sigma(2j+1)C^2 S_{\text{adding}} + \Sigma C^2 S_{\text{removing}}]$$

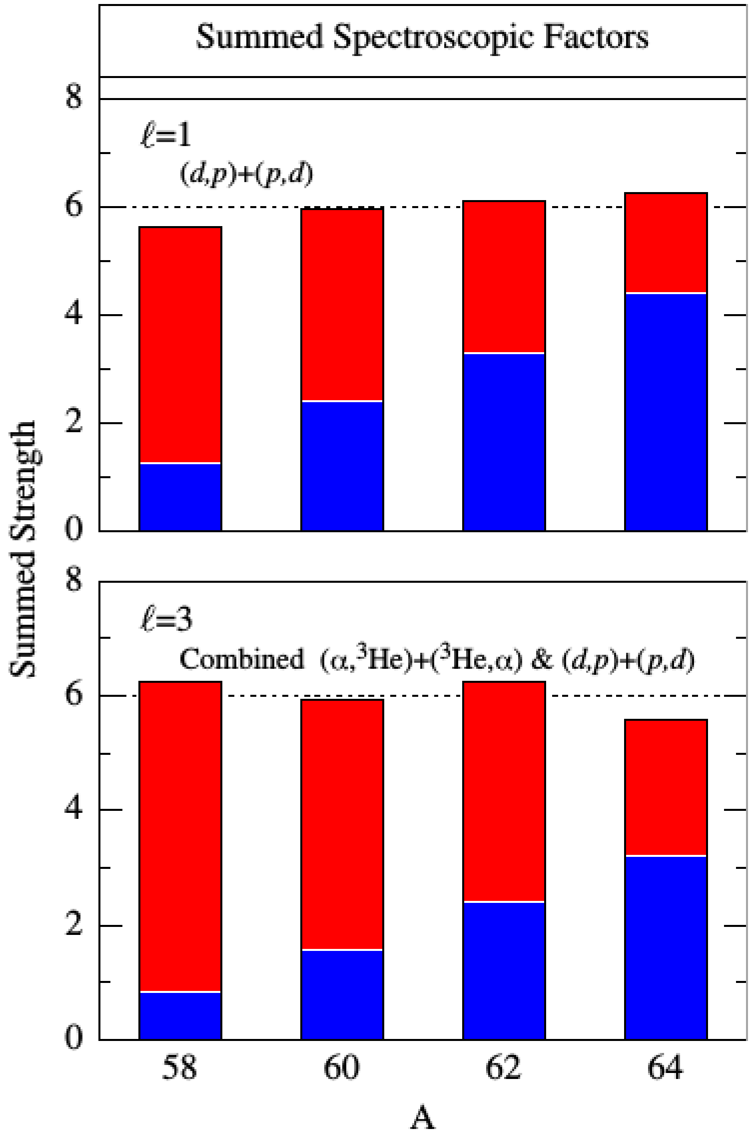
Results of the Normalization for Z = 28

The Macfarlane and French SUM RULES are shown to be robust!!

Neutron adding strength

Neutron removal strength

Nucleus	$N_{\ell=1}$	$N_{\ell=3}$	$N_{\ell=3,\alpha}$
^{58}Ni	0.527	0.528	0.518
^{60}Ni	0.548	0.503	0.464
^{62}Ni	0.558	0.554	0.471
^{64}Ni	0.566	0.480	0.433
Mean	0.550(15)	0.517(28)	0.471(30)



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Similar normalizations for each isotope!!

Independent of orbital!!

< 10 % deviation

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Good agreement for the normalizations across the different methods
< 20% deviation

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Mean	0.550(15)	0.517(28)	0.471(30)

Deuteron	Proton	Bound state	Normalization
[12], fixed	[13], fixed	[15]	0.492 ± 0.020
[16]	[13], fixed	[15]	0.646 ± 0.041
[17]	[13]	[15]	0.568 ± 0.037
[12]	[13]	[15]	0.550 ± 0.015
[18]	[19]	[15]	0.572 ± 0.051
[12]	[13]	[13]	0.475 ± 0.018
[12]	[14]	[14]	0.561 ± 0.022

Different optical parameters for (d,p)(p,d) *l* = 1 still shows consistency

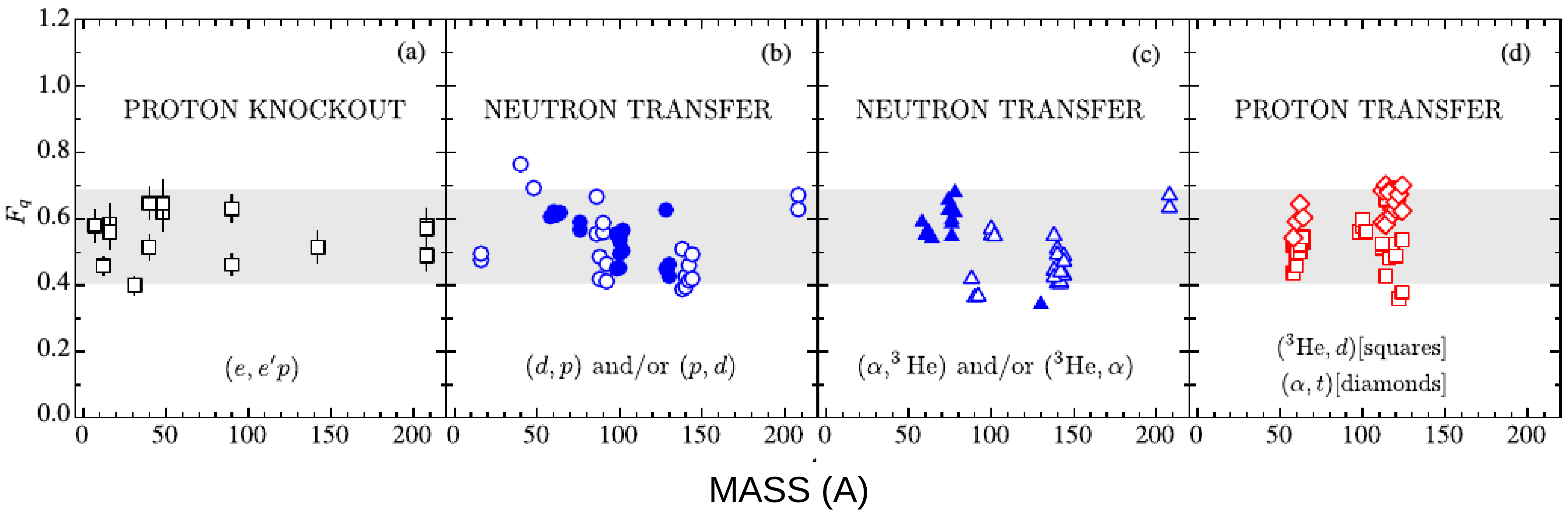
Global Consistency of the Normalization

Reaction, ℓ transfer	Number of determinations	F_q	rms spread
$(e,e'p)$, all ℓ	16	0.55	0.07
(d,p), (p,d), $\ell = 0$–2	40	0.53	0.09
(d,p), (p,d), $\ell = 0$–3	46	0.53	0.10
$(\alpha,{}^3\mathrm{He})$, $({}^3\mathrm{He},\alpha)$, $\ell = 4$–7	26	0.50	0.09
$(\alpha,{}^3\mathrm{He})$, $({}^3\mathrm{He},\alpha)$, $\ell = 3$–7	34	0.52	0.09
$({}^3\mathrm{He},d)$, $\ell = 0$–2	18	0.54	0.10
$({}^3\mathrm{He},d)$, $\ell = 0$–4	26	0.54	0.09
(α,t), $\ell = 4$–5	14	0.64	0.04
(α,t), $\ell = 3$–5	18	0.64	0.04
All transfer data[a]	124	0.55	0.10

[a]Rows 3, 5, 7, and 9.

$$F_q \equiv \frac{1}{(2j+1)}\left[\Sigma\left(\frac{\sigma_{\exp}}{\sigma_{\mathrm{DW}}}\right)_j^{\mathrm{add}} + \Sigma\left(\frac{\sigma_{\exp}}{\sigma_{\mathrm{DW}}}\right)_j^{\mathrm{rem}}\right],$$

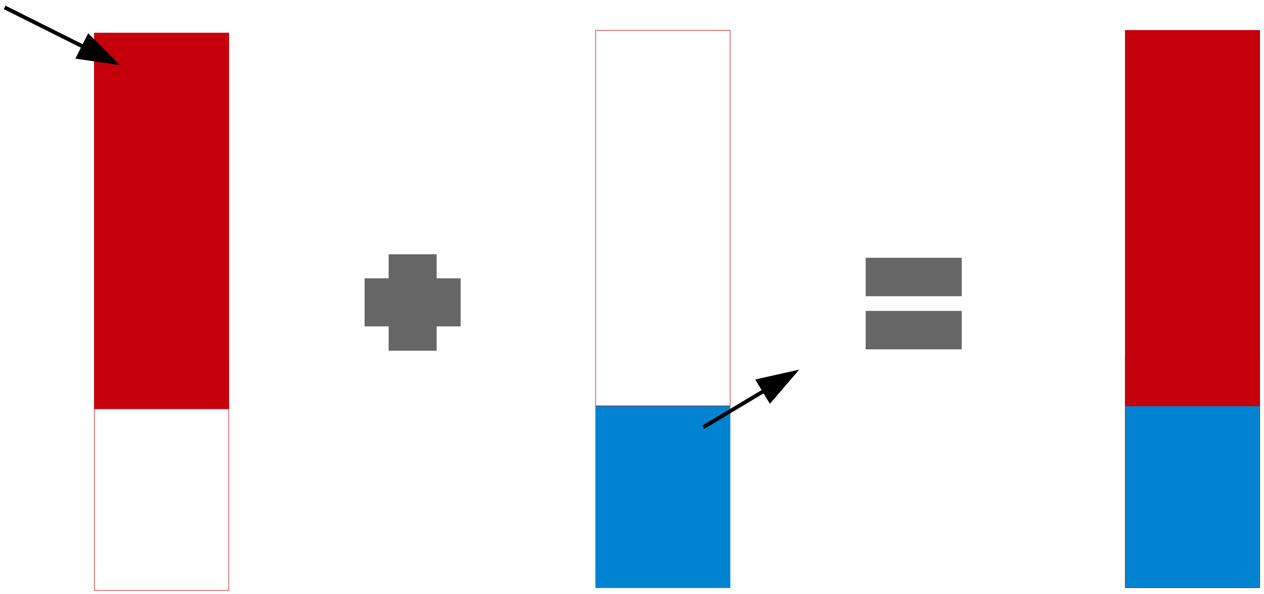
→ same as N



Success of Sum Rules Procedures

- Systematic test of the Macfarlane and French sum rules was carried out on the stable even-even Ni isotopes
- Care was taken in the experiment to extract reliable and consistent absolute cross sections
- Common procedures were used to reduce the cross sections to spectroscopic factors (reduced cross sections)
- Consistent values for the normalization value (N) using the sum rules was obtained for each of the different targets
- Value of the normalization is common across all nuclei (N ~ 0.4 – 0.7) using consistent optical model parameters

Extract normalization value using sum rules for each target



Orbital (njl)

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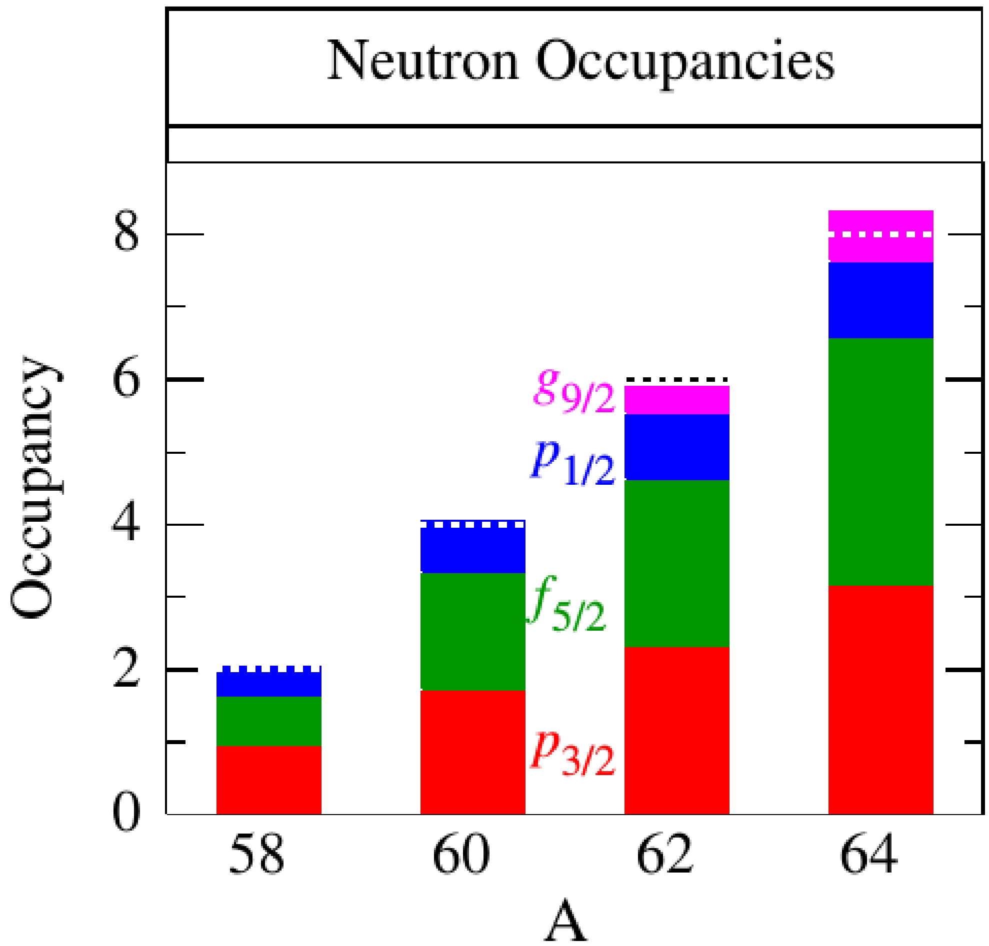
Normalization [for single orbital]

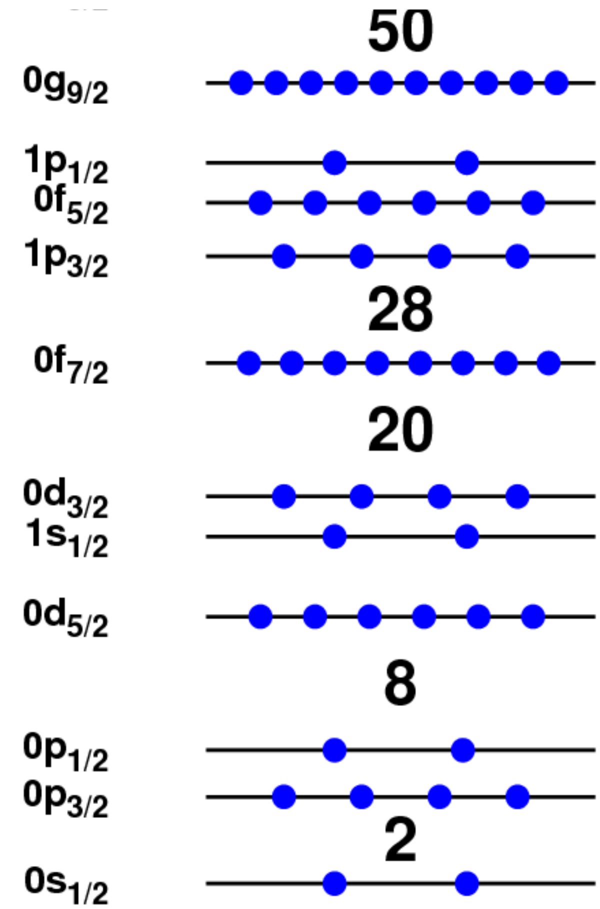
Macfarlane and French, Rev. Mod. Phys. 32, 567 (1960)

Determination of Neutron Occupancies

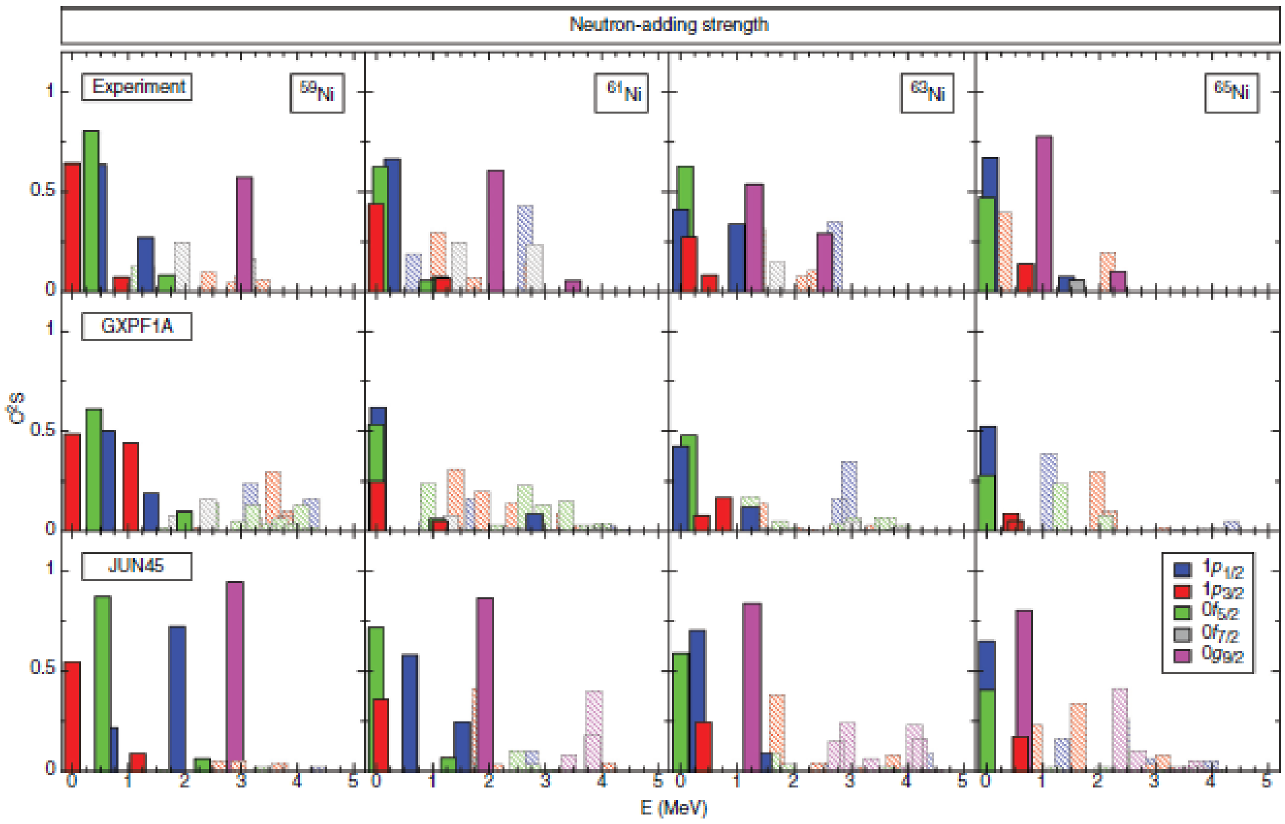
Sum only the neutron removal strength (OR 2j+1 – adding strength)

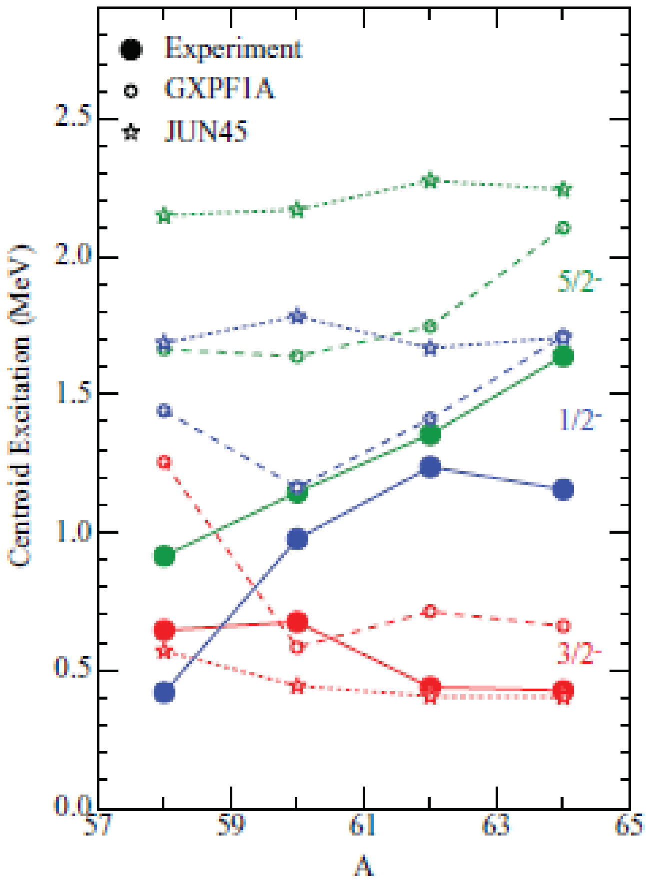


N = 30 32 34 36



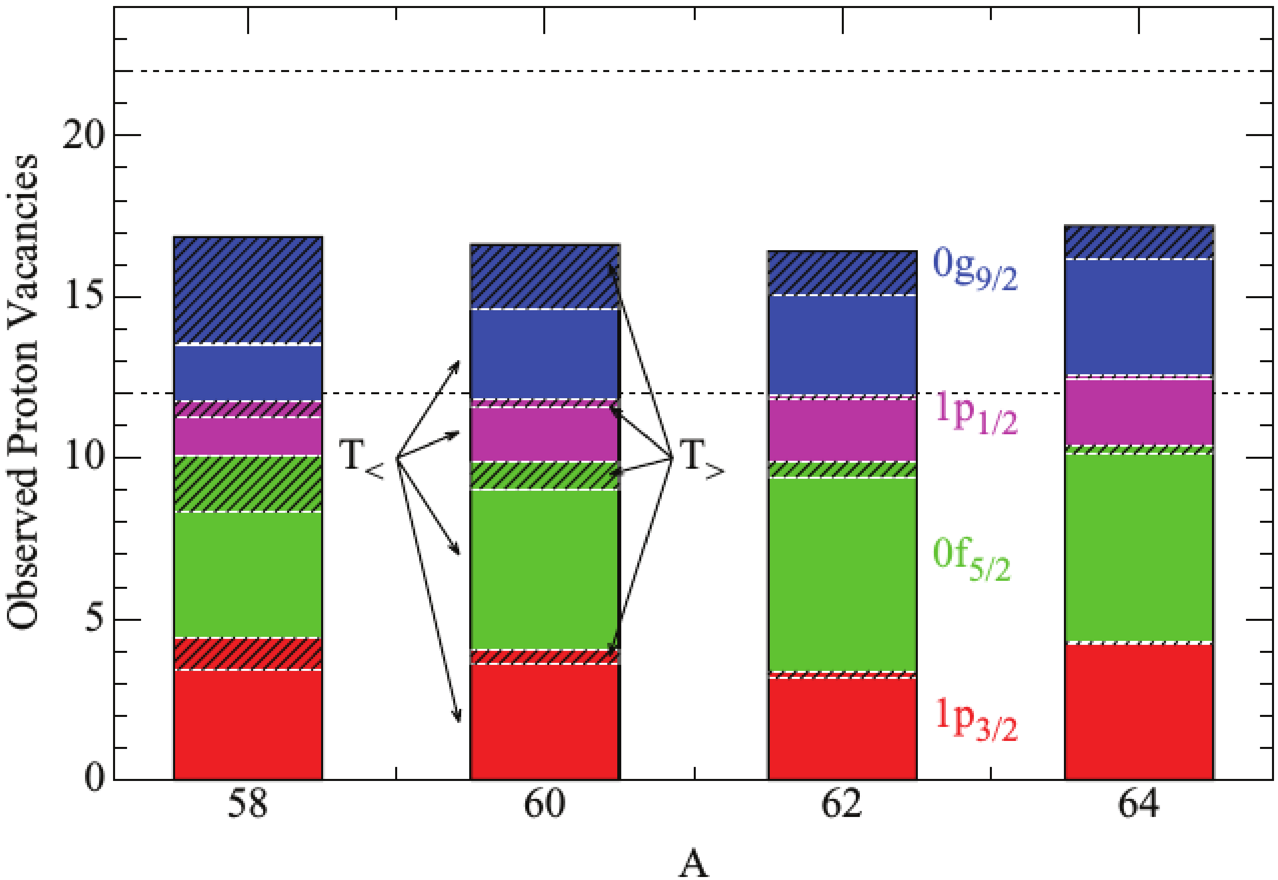
Applications: Single-Neutron Strength and Single-Neutron Energy Centroids

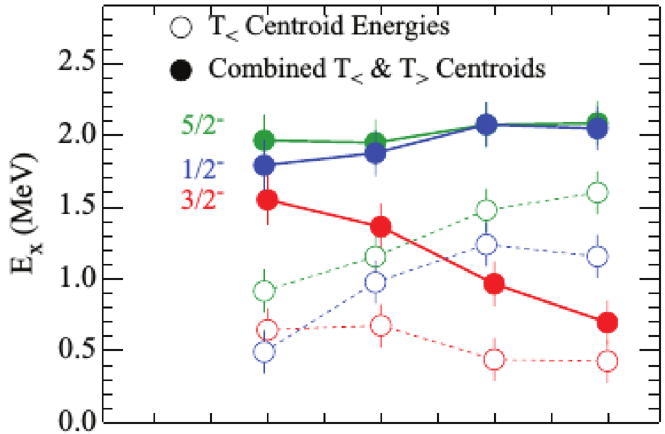


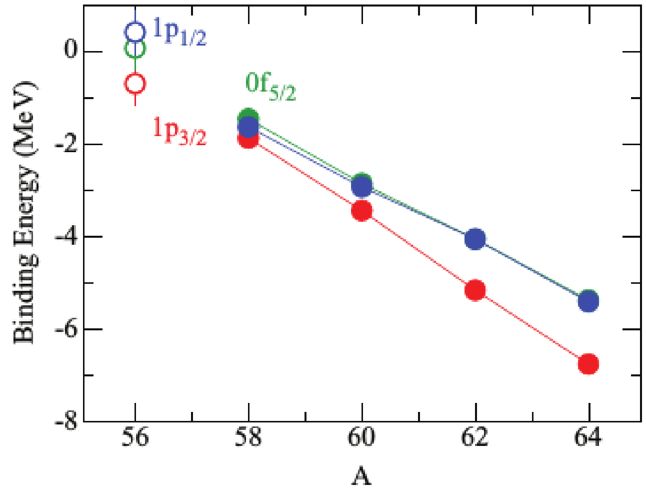


Protons Occupancies and Centroids

- Only measured proton adding reactions → additional assumptions made e.g., robust Z = 28 shell closure







Question

- Who are the defending National Champions in College Football? (Hint: Their quarterback also won the Heisman Trophy!!)

A) Michigan State University Spartans

B) University of Connecticut Huskies

C) Florida State University Seminoles

D) A school from the SEC

E) If football = soccer

C) Florida State University Seminoles!!

Thank You!

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