EBSS-13 – nuclear astrophysics

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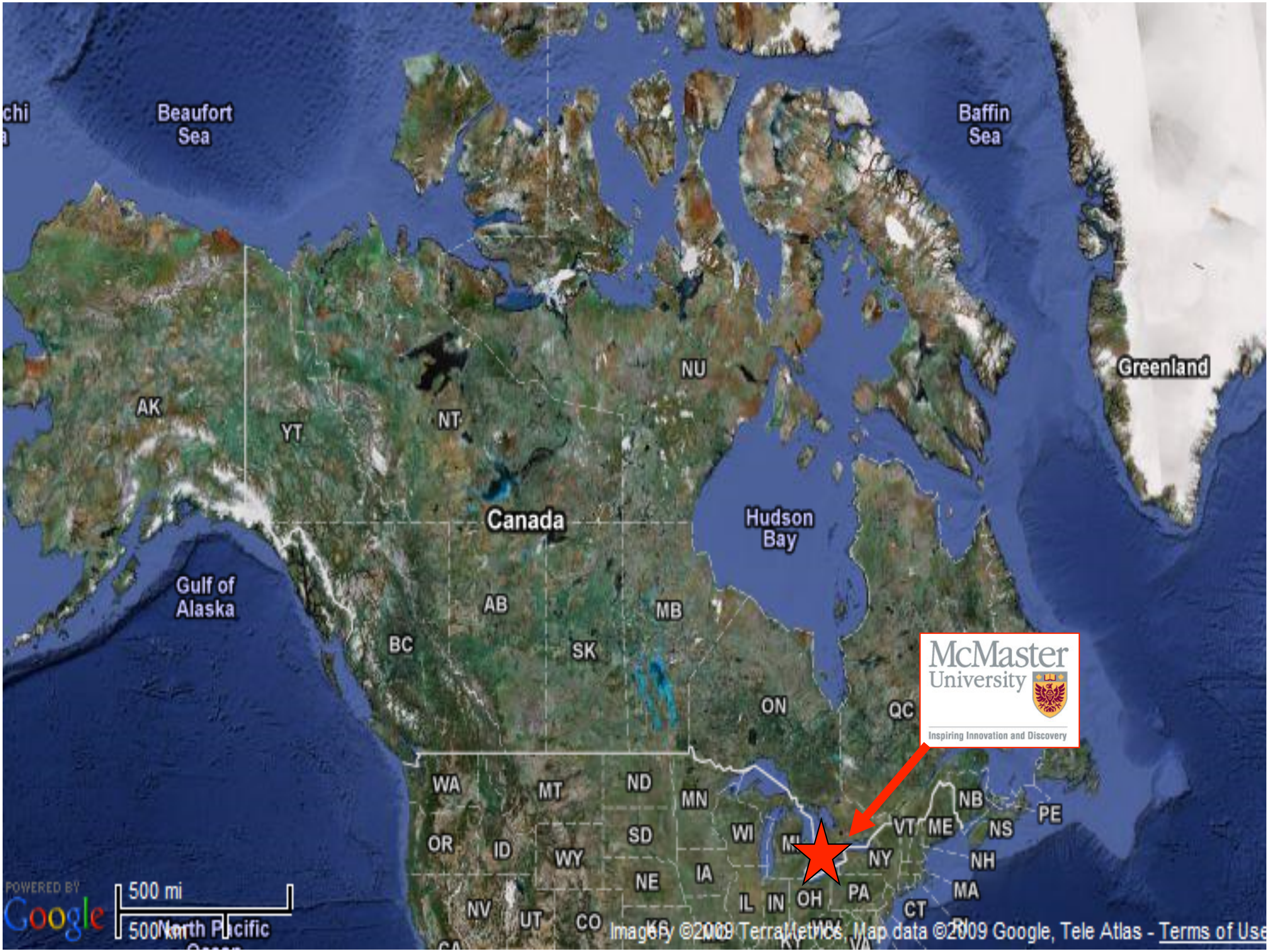

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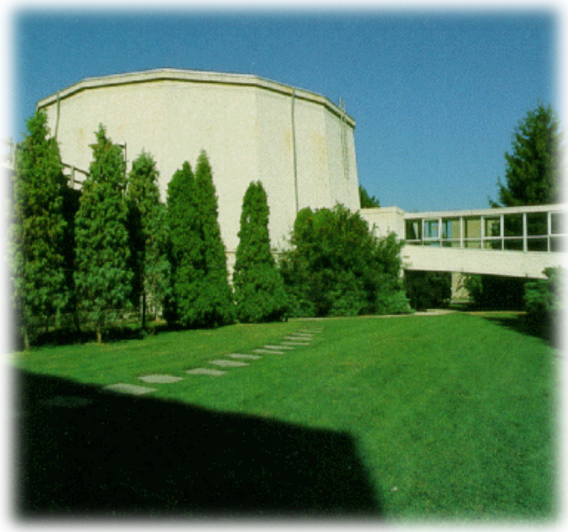


nuclear astrophysics: goals

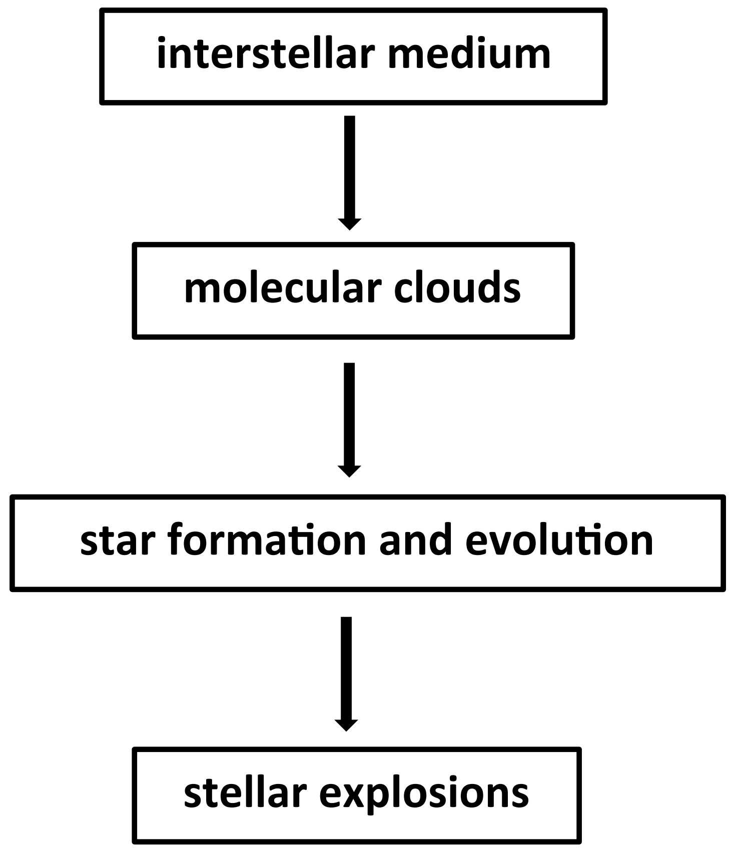
- **seek to understand:**
  - **energy generation in stars**
  - **origin of the elements**

[Burbidge, Burbidge, Fowler and Hoyle (1957)]
[Cameron (1957)]

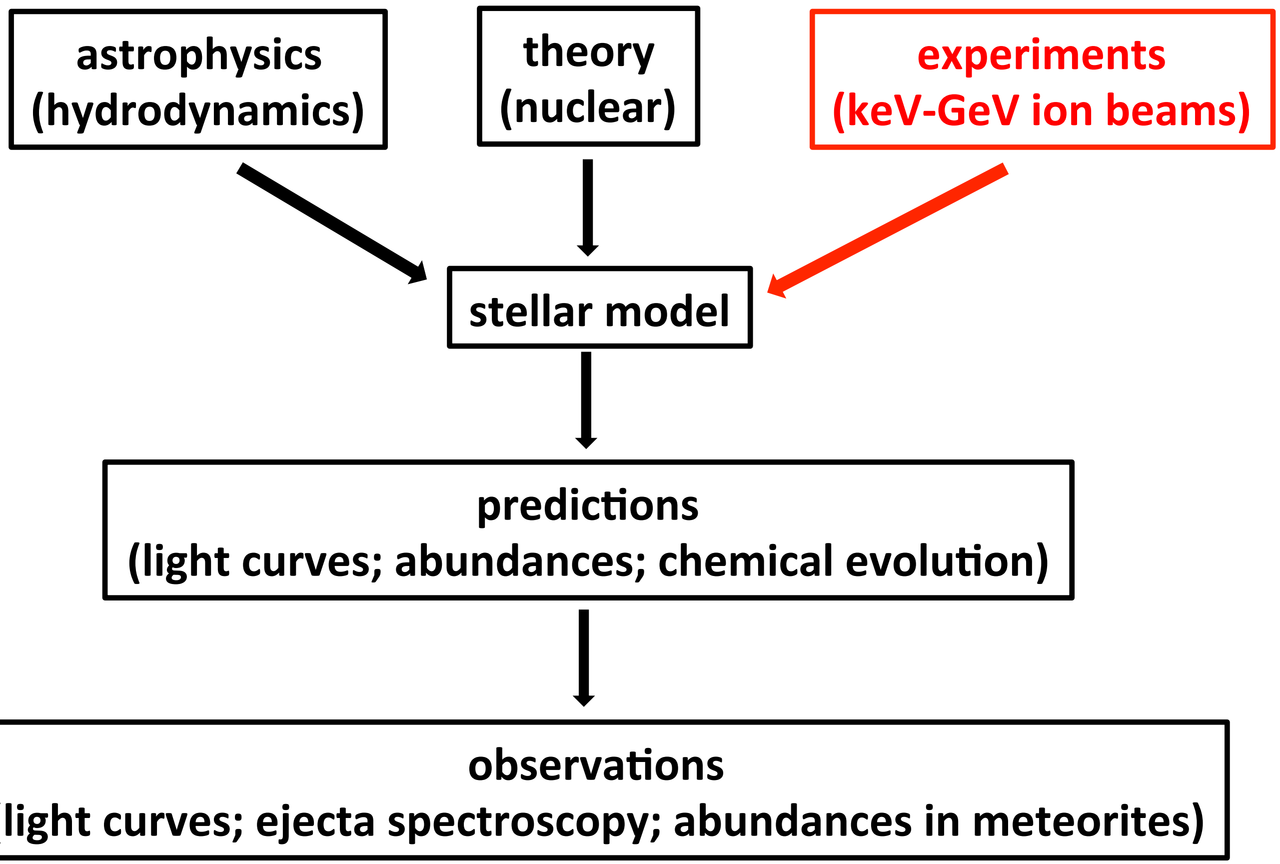
[recent review: J. José and C. Iliadis, Rep. Prog. Phys. (2011)]

[textbooks: D. Clayton, *Stellar Evolution and Nucleosynthesis* (1983)
C. Rolfs and W. Rodney, *Cauldrons in the Cosmos – Nuclear Astrophysics* (1988)
C. Iliadis, *Nuclear Physics of Stars* (2007)]

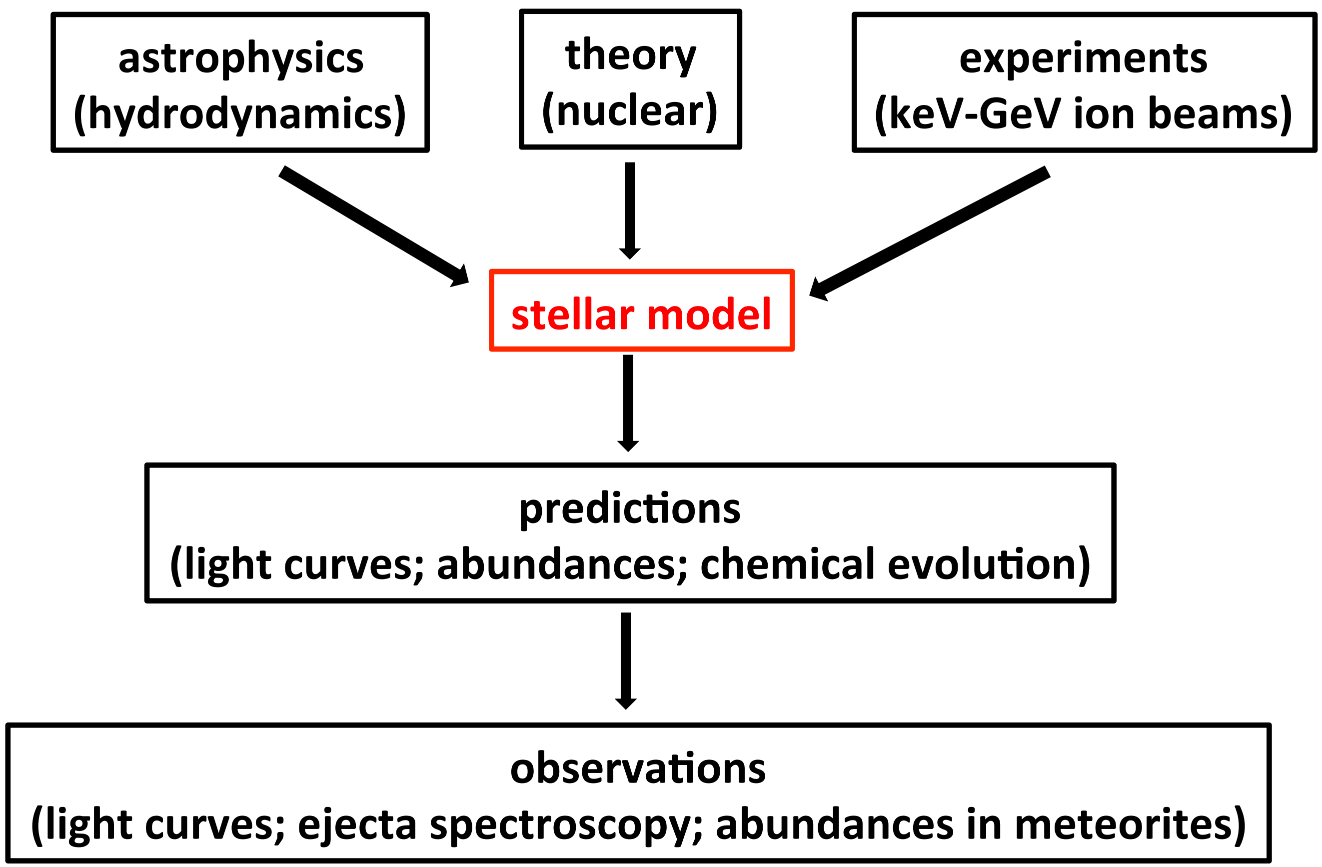
nuclear astrophysics: big picture



nuclear astrophysics: progress



nuclear astrophysics: progress



stellar evolution: hydrostatic "quiescent" burning

$$\frac{dP(r)}{dr} = -\frac{Gm(r)\rho(r)}{r^2}$$

[hydrostatic equilibrium]

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r)$$

[mass continuity]

$$\frac{dT(r)}{dr} = -\frac{3\kappa(r)\rho(r)L(r)}{(4\pi r^2)(16\sigma)T^3(r)}$$

[radiative diffusion]

$$\frac{dL(r)}{dr} = 4\pi r^2 \varepsilon(r)$$

[thermal equilibrium]

stellar evolution: hydrostatic “quiescent” burning

$$\frac{dP(r)}{dr} = -\frac{Gm(r)\rho(r)}{r^2}$$ **[hydrostatic equilibrium]**

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r)$$ **[mass continuity]**

$$\frac{dT(r)}{dr} = -\frac{3\kappa(r)\rho(r)L(r)}{(4\pi r^2)(16\sigma)T^3(r)}$$ **[radiative diffusion]**

$$\frac{dL(r)}{dr} = 4\pi r^2 \varepsilon(r)$$ **[thermal equilibrium]**

plus: ideal gas law, Stefan –Boltzmann law, Kramers opacity approximation, and

$$\varepsilon(r) = \varepsilon_o \rho^2(r) T^{\nu}(r)$$ **[energy generation]**

stellar evolution: hydrostatic "quiescent" burning

- $\varepsilon(r)$ for "main sequence" stars: 4p → ^{4}He + ~26 MeV
  - <u>proton-proton chains</u> (Sun):
    - $p + p \rightarrow d + e^{+} + \nu_e$: slowest reaction
    - stellar lifetime ~ several billion years
    - $\nu \sim 4$
  - <u>CNO cycles</u> (Sirius A, ~ 2 M_{sun}):
    - $^{14}N(p,\gamma)^{15}O$: slowest reaction
    - stellar lifetime < a billion years
    - $\nu \sim 20$

stellar evolution: helium burning

- **core hydrogen exhausted → helium core contracts**
  - **ρ_{core} and T_{core} increase**
  - **core helium burning is ignited:**
    - **3α reaction**
    - **$^{12}C(\alpha,\gamma)\,^{16}O$**

beyond core helium burning in massive stars

- core: helium → carbon → oxygen → neon → silicon

  ⟶ [temperature]

  - $T_{core} \sim 10^9$ K : nuclear reactions in equilibrium with their inverses

    → isotopic abundances:

    Nuclear Statistical Equilibrium (NSE)

massive stars, cont'd: nuclear statistical equilibrium

- core: silicon → $T_{core} \sim 10^9$ K

  ∴ α-captures on ^{28}Si → heavier nuclei

- (thermodynamic) equilibrium: $$\frac{n_A n_\alpha}{n_{[A+\alpha]}} \propto \exp(-\Delta Q/kT)$$

  n = number density
  ΔQ = energy change in α-capture

∴ $\Delta Q > 0$ → [A + α] favored

$\Delta Q < 0$ → A , α favored

massive stars, cont'd: nuclear statistical equilibrium

- α-captures in NSE: $\Delta Q > 0 \rightarrow [A + \alpha]$ favored

  $\Delta Q < 0 \rightarrow A\ ,\ \alpha$ favored

- since B.E./A $\sim$ max near A = 56:

  - $\Delta Q > 0$ for $A < 56$ (and $\Delta Q < 0$ for $A > 56$)

  $\therefore$ equilibrium takes light nuclei toward A = 56

  $\rightarrow$ IRON PEAK in abundance distribution

  Nuclear energy source exhausted $\rightarrow$ core collapse $\rightarrow$
  $\rightarrow$ explosion (type II supernova)

rare isotopes in stars: supernovae

[Cassiopeia A]

- Type II, Type Ia

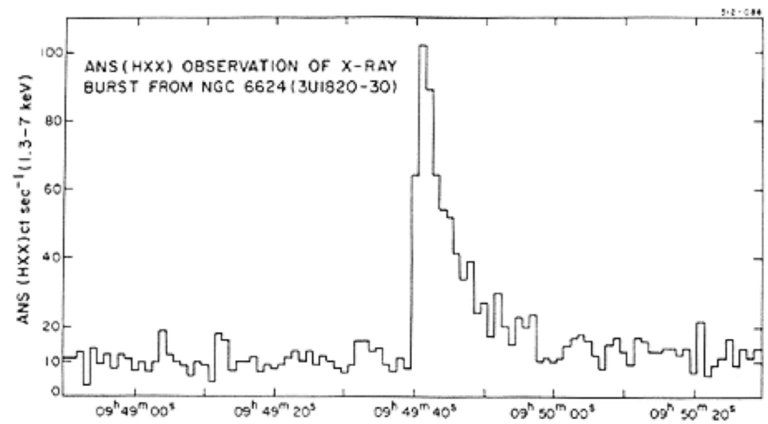
- important <u>nuclear physics</u> :

  r-process: neutron captures

  weak interactions: *e.g.*, electron captures

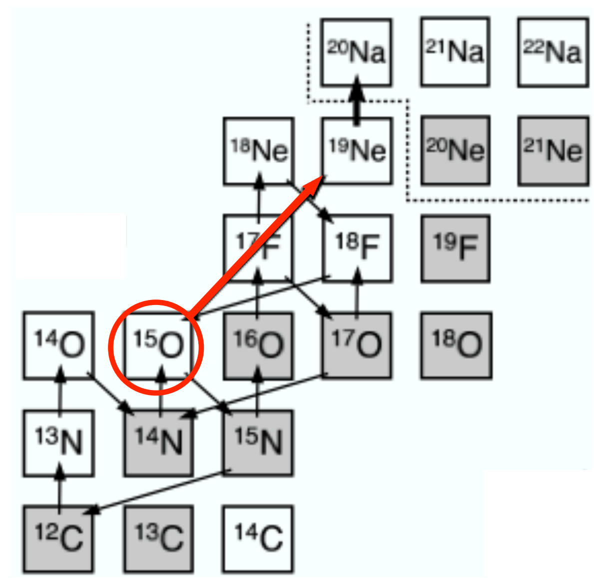
rare isotopes in stars: type I x-ray bursts

- <u>model</u>:
  - binary star system
  - accretion on <u>neutron star</u>
  - thermonuclear runaway



- <u>observations</u>: light curves

- <u>research areas</u>:
  - Breakout from the Hot-CNO cycles
  - rp-process: path, endpoint, synthesis
  - αp-process → key reactions



- <u>experiments</u>: proton-rich rare isotopes
  - (p,γ) and (α,p) reactions
  - mass measurements

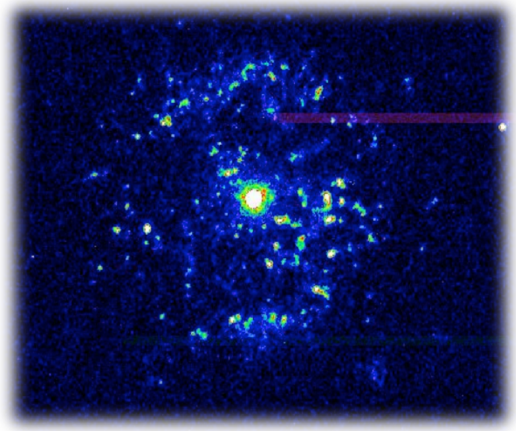
rare isotopes in stars: classical novae

- <u>models</u>:
  - binary star system
  - accretion on <u>white dwarf</u>
  - thermonuclear runaway

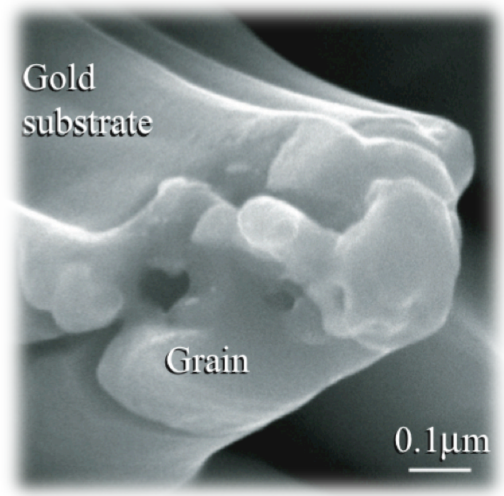
- <u>observations</u>: ejecta spectroscopy
  presolar meteoritic grains

- <u>research areas</u>:
  - Ne-Na, Mg-Al cycles
  - reactions affecting synthesis of:
    - γ-emitters (*e.g.*, ^{18}F, ^{22}Na, ^{26}Al)
    - isotopes in meteoritic grains
    - elements in ejecta

- <u>experiments</u>: proton-rich rare isotopes
  - (p,γ) and (p,α) reactions



[Nova Pyxidis]



rare isotopes in stars: gamma-ray astronomy (^{26}Al, ^{44}Ti)

[Cassiopeia A]

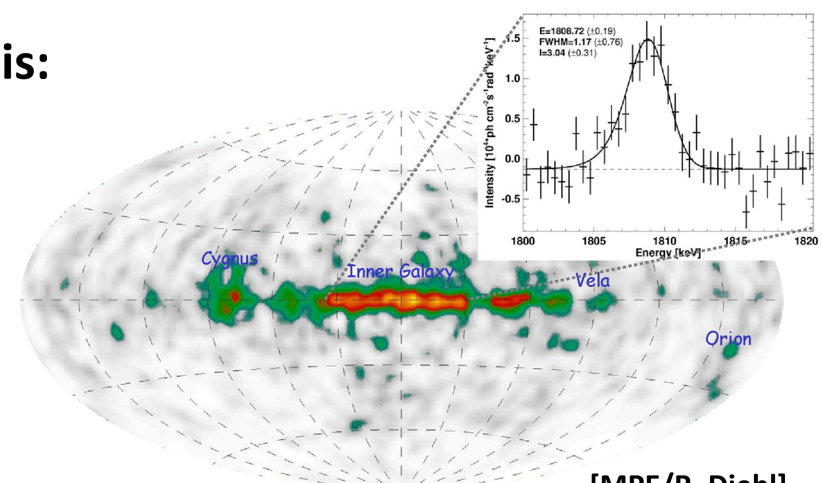
- <u>observations</u>: γ-ray emission from ^{26}Al decay
    - diagnostic of ongoing nucleosynthesis
    - constraint on galactic chemical evolution

- <u>models</u>: need ^{26}Al yield predictions for different stars (e.g., supernovae, classical novae, AGB stars)

- important <u>reactions</u> affecting ^{26}Al synthesis:

  ^{25}Al(p,γ)^{26}Si

  ^{26}Al(p,γ)^{27}Si



[MPE/R. Diehl]

nucleosynthesis in the lab

- **explosive hydrogen/helium burning:**

T ~ 0.1 – few GK

→ E_{cm} ~ 100 keV – few MeV

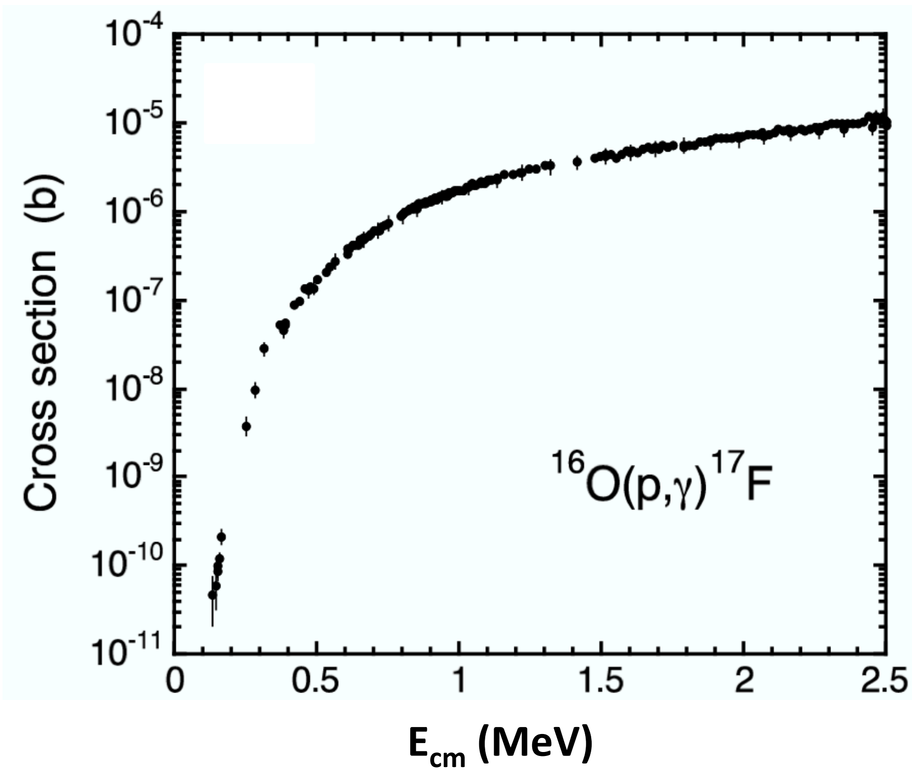
- ***unstable* nuclei are important**

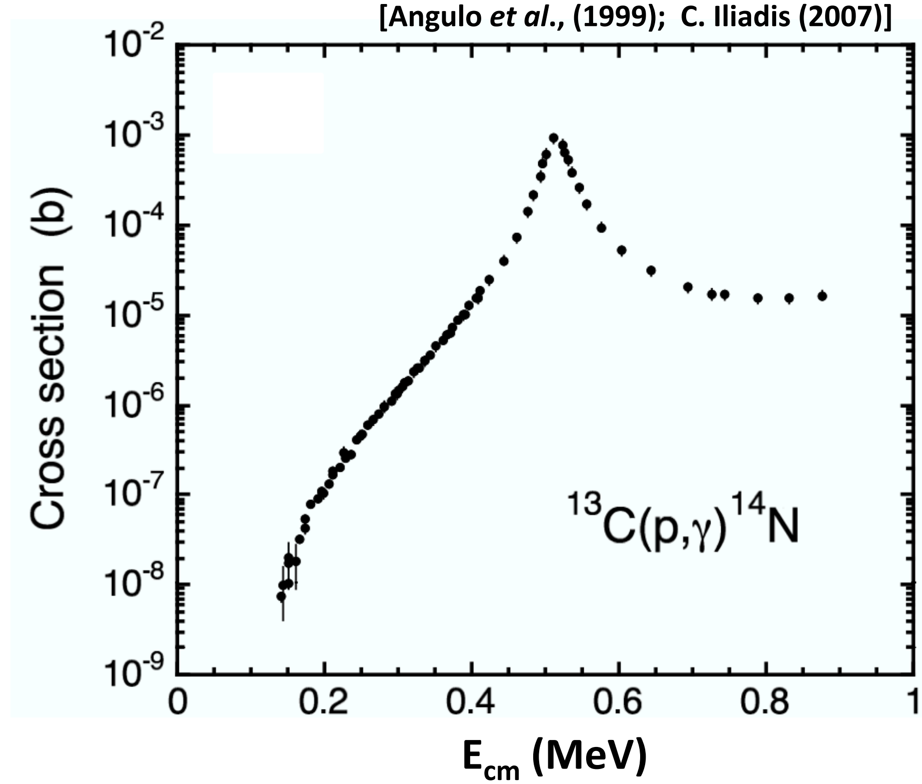
- **goal: *cross-sections → thermonuclear reaction rates***

- **direct and indirect approaches**

nuclear reactions: cross section

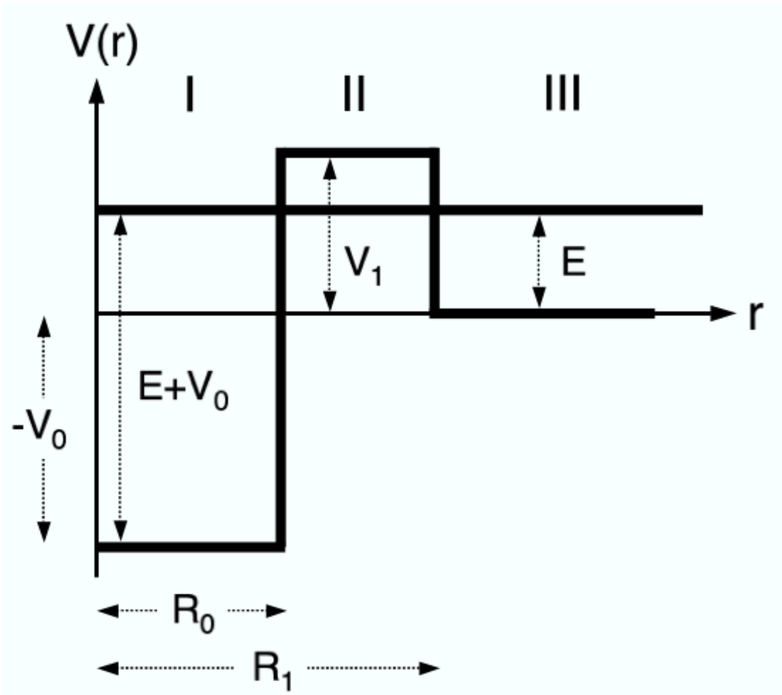
[Angulo *et al*., (1999); C. Iliadis (2007)]





- **drastic fall at low energies**
- **resonance peaks**

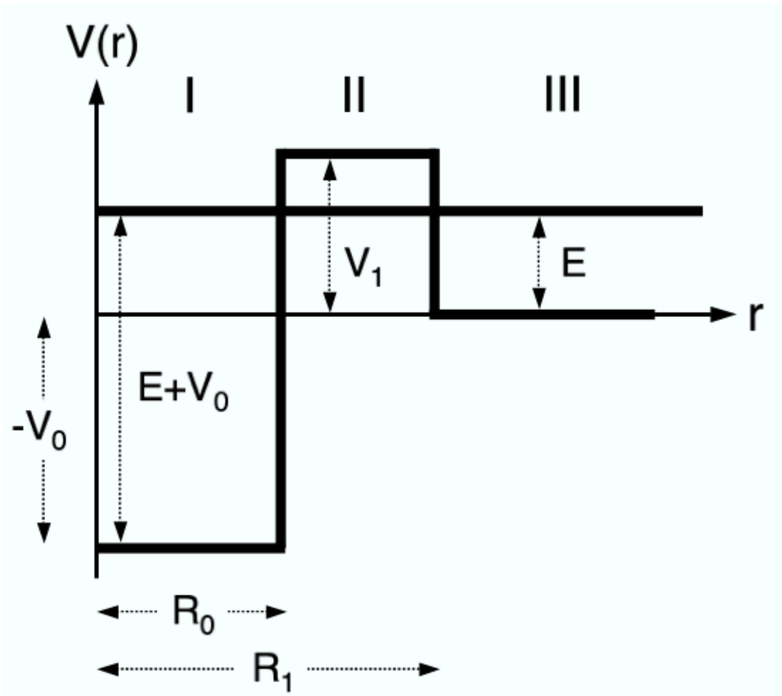
nuclear cross section: features



[figure from C. Iliadis (2007)]

- **steep drop and peaks: quantum mechanics**
- **Schrödinger's equation:**
  → wavefunctions
- **continuity of wavefunction and derivative:**
  → wavefunction amplitudes

nuclear cross section features



[C. Iliadis (2007)]

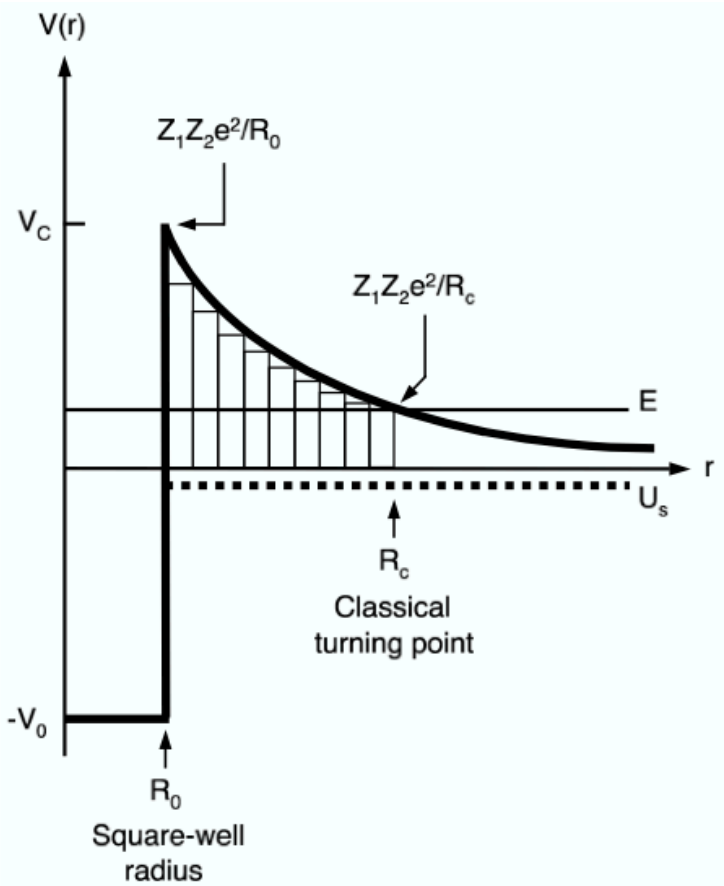
- **tunneling**:

$$T \approx \exp\left(-\frac{2}{\hbar}(R_1 - R_0)\sqrt{2m(V_1 - E)} \right)$$

(transmission coefficient for s-wave neutrons)

- **resonances**:
  - **good matching of (radial) wavefunction at boundaries**
  - **large amplitude of wavefunction inside**

cross sections: Coulomb barrier



[figure from C. Iliadis (2007)]

- **transmission coefficient:**

$$T \approx \exp\left(-\frac{2}{\hbar} \int_{R_0}^{R_c} \sqrt{2m[V(r)-E]} \right)$$

- **for s-waves and low energies:**

$$T \approx \exp\left(-\frac{2\pi}{\hbar} \sqrt{\frac{m}{2E}} Z_1 Z_2 e^2 \left[1 + \frac{2}{3\pi} \left(\frac{E}{V_C} \right)^{3/2} \right] + \ldots \right)$$

- **leading term:**

$$T \approx \exp\left(-\frac{2\pi}{\hbar} \sqrt{\frac{m}{2E}} Z_1 Z_2 e^2 \right) \quad \equiv \exp(-2\pi\eta)$$

[Gamow factor]

cross sections: resonances

- **Breit-Wigner formula:**

partial widths of entrance and exit channels

$$\sigma_{BW}(E) = \frac{\lambda^2}{4\pi} \frac{(2J+1)}{(2j_1+1)(2j_2+1)} \frac{\Gamma_a \Gamma_b}{(E_r - E)^2 + \Gamma^2/4}$$

resonance energy **total width**

- **Applications:**
  extract J^{π} , E_r , Γ's from data
  parameterize "narrow resonances" for reaction rates

cross sections: partial widths

- **partial width: probability for formation/decay of resonance**

$$\Gamma_p = 2\frac{\hbar^2}{mR^2} P_\ell \, C^2 S \, \theta_p^2$$

P_l **: penetration factor**

C^2S**: spectroscopic factor (with CG coefficient)**

θ^2 : single-particle reduced width

thermonuclear reactions

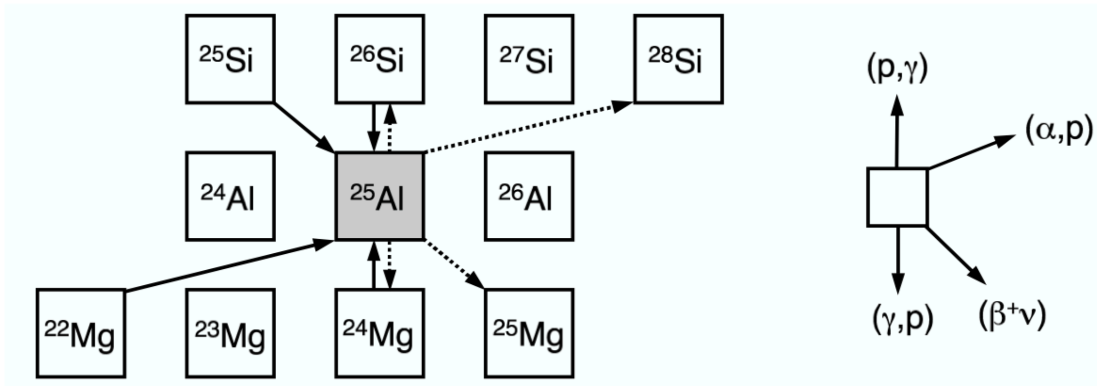
- **reaction rate:** $r_{12} = N_1\, N_2 \int_0^\infty \upsilon\, P(\upsilon)\, \sigma(\upsilon)\, d\upsilon \equiv N_1\, N_2 \langle \sigma\upsilon \rangle_{12}$

- **stellar plasma: use Maxwell-Boltzmann distribution:**

$$\langle \sigma v \rangle_{12} = \left(\frac{8}{\pi\, \mu_{12}} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty E\, \sigma(E) \exp(-\frac{E}{kT})\, dE$$

(thermonuclear reaction rate)

thermonuclear reactions: network



$$
\begin{aligned}
\frac{d(N_{^{25}\mathrm{Al}})}{dt} =& N_{\mathrm{H}} N_{^{24}\mathrm{Mg}} \langle \sigma v \rangle_{^{24}\mathrm{Mg(p,}\gamma)} + N_{^{4}\mathrm{He}} N_{^{22}\mathrm{Mg}} \langle \sigma v \rangle_{^{22}\mathrm{Mg}(\alpha,\mathrm{p})} \\
&+ N_{^{25}\mathrm{Si}} \lambda_{^{25}\mathrm{Si}(\beta^{+}\nu)} + N_{^{26}\mathrm{Si}} \lambda_{^{26}\mathrm{Si}(\gamma,\mathrm{p})} + \ldots \\
&- N_{\mathrm{H}} N_{^{25}\mathrm{Al}} \langle \sigma v \rangle_{^{25}\mathrm{Al(p,}\gamma)} - N_{^{4}\mathrm{He}} N_{^{25}\mathrm{Al}} \langle \sigma v \rangle_{^{25}\mathrm{Al}(\alpha,\mathrm{p})} \\
&- N_{^{25}\mathrm{Al}} \lambda_{^{25}\mathrm{Al}(\beta^{+}\nu)} - N_{^{25}\mathrm{Al}} \lambda_{^{25}\mathrm{Al}(\gamma,\mathrm{p})} - \ldots
\end{aligned}
$$

[from C. Iliadis (2007)]

thermonuclear reactions: Gamow peak

- **substitute** $\sigma(E) \equiv \frac{1}{E} \exp(-2\pi\eta)\, S(E)$

into $\langle \sigma v \rangle_{12} = \left(\frac{8}{\pi \mu_{12}} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty E\, \sigma(E) \exp(-\frac{E}{kT})\, dE$

$$\Rightarrow \quad \langle \sigma v \rangle \sim \int_0^\infty S(E) \boxed{\exp(-2\pi\eta) \exp(-E/kT)}\, dE$$

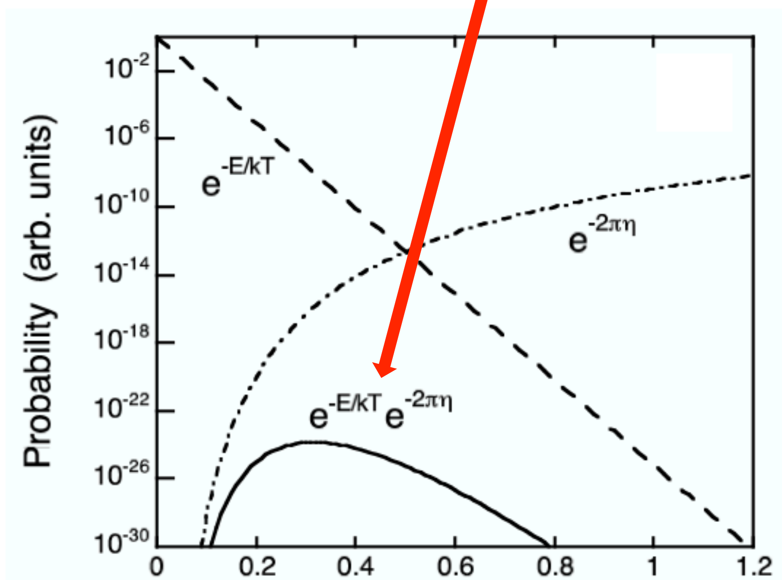
"Gamow peak"

energy range within which most reactions occur

[*S(E)* : astrophysical S-factor]

thermonuclear reactions: Gamow peak

$$\langle \sigma v \rangle \sim \int_0^\infty S(E) \exp(-2\pi\eta) \exp(-E/kT)\, dE$$



Energy (MeV)

$^{12}C(\alpha,\gamma)^{16}O$
(helium burning)
T = 0.2 GK

[figure from Iliadis (2007)]

thermonuclear reaction: narrow resonances

$$\langle \sigma v \rangle \sim \int_0^\infty E\,\underbrace{\sigma(E)}\,\exp(-\frac{E}{kT})\,dE$$

<u>Breit-Wigner formula</u>:

$$\sigma_{BW}(E) \sim \frac{\omega \boxed{\Gamma_a\,\Gamma_b}}{(E_r - E)^2 + (\Gamma/2)^2}$$

partial widths of entrance and exit channels

resonance energy

total width

$$\Rightarrow \quad \langle \sigma v \rangle \sim \underbrace{\exp(-E_r/kT)}\,\underbrace{\omega\frac{\Gamma_A\Gamma_B}{\Gamma}}$$

resonance energy: needs to be measured precisely

"resonance strength" $\omega\gamma$

[<u>broad resonances</u>: widths are energy-dependent → calculate reaction rate analytically]